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Extending the economic framework to model correlations between PD, LGD, and EAD

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Abstract

This study examines the extension of the economic framework to correlations between PD, LGD, and EAD. We build on a framework that has already been used to figure out and adjust the relationships between loan portfolios' Probability of Default (PD), Loss Given Default (LGD), and Exposure at Default (EAD). Our analysis explores the implications of incorporating these correlations in portfolio losses, arguing that this structure enables institutions to apply forward-looking correlation models to assess the likelihood of obligor credit quality deterioration, commonly referred to as a significant increase in credit risk (SICR). According to International Financial Reporting Standards (IFRS)-9 regulations, the estimation of SICR and forward-looking information should not entail excessive cost or effort. In line with this principle, we contend that only a limited number of inputs are necessary to implement this robust framework, which allows users to evaluate meaningful forward-looking correlations, identify obligors likely to experience SICR, and ultimately measure a more accurate Expected Credit Loss (ECL). The adoption of this approach will allow institutions to better understand their credit risk and better assess their credit risk practices while adhering to regulatory requirements.

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1. Introduction

During the financial crisis of 2008, the International Accounting Standard Board (IASB) and Financial Accounting Standard Board (FASB) collaborated to revamp accounting standards, aiming for a more effective and streamlined ECL framework. This effort resulted in the release of the IFRS 9 in 2014 (IFRS, 2014). IFRS 9 addresses how organisations should recognise their financial assets and liabilities in their financial statements, emphasising an ECL framework for identifying impairment. ECL quantification typically involves assessing three key components: the probability of default (PD), loss given default (LGD), and exposure at default (EAD), and it mandates that ECL models consider both current and forecasted macroeconomic conditions to assess credit loss. This approach facilitates the calculation of forward-looking impairment estimates.

Existing regulatory frameworks do not adequately account for the correlations between PD, LGD, and EaD, leading to potential underestimations of capital requirements. Basel credit risk regulations require the use of a downturn LGD (worst recorded LGD over a period of seven years) to ensure that banks are adequately prepared for potential losses during economic downturns. This requirement stems from the understanding that credit risk is inherently cyclical, with default rates and losses typically increasing during periods of economic stress. In such times, not only does the likelihood of default rise, but recoveries for defaulted exposures often diminish due to weakened economic conditions. Incorporating a downturn in LGD allows the regulations to provide a more conservative and realistic estimate of potential losses that banks might face during adverse economic scenarios. Aside from protecting depositors, downturn LGDs also help regulators keep the financial system stable by making sure banks have enough capital to cover possible losses even in the worst economic times. This lowers the risk of bank failure and protects depositors. Requiring banks to use downturn LGD also promotes a forward-looking perspective in risk management by encouraging financial institutions to consider not only the current state of the economy but also the potential for future adverse credit risk conditions. This anticipatory stance helps banks build a more comprehensive risk profile and prepare for a range of economic scenarios, enhancing their overall resilience.

Despite the regulatory requirements, an understanding of the true correlations between critical variables (PD, LGD, and EaD) is necessary for accurate capital estimations. Recent novel frameworks explicitly model EaD as a stochastic variable and capture the correlations between all three variables (PD, LGD, and EaD). These kinds of models have been shown to give more accurate estimates of credit losses in terms of ECL under IFRS-9 and events that happen at the tail end of the Credit Value-at-Risk (VaR) scale. By modelling EaD in a correlated manner with PD and LGD, such frameworks enable lenders to model the increase in funding requirements during downturns as borrowers draw down on their credit lines and provide a more comprehensive approach to estimating capital requirements by calibrating these correlations based on historical data.

Using such a framework, we model the correlation structure for wide input parameter ranges to explore the nature and magnitude of credit losses under different scenarios. In this way, intriguing relationships can be discerned at parameter extremities, which might unveil areas of concern for lenders.

The remainder of this article proceeds as follows: Section 2 provides a literature survey of work undertaken in the field of credit parameter correlations, limitations of such approaches, and possible mitigations. Section 3 sets out the data (simulations were deliberately employed to understand wide ranges of parameter inputs) and the governing mathematical methodology, while Section 4 reports on the results obtained and opines on possible reasons for the observed results. Section 5 concludes.

2. Literature Survey

2.1. PD and EAD Correlations

There is less research on PD-EAD correlations, but the existing evidence suggests a positive relationship. Roesch and Scheule (2010) found a correlation of 0.3 for a portfolio of retail loans, while Jacobs Jr and Karagozoglu (2011) estimated a correlation of 0.2 for a sample of corporate loans. The positive PD-EAD correlation can be attributed to the tendency of borrowers to increase their utilisation of credit lines as the probability of default rises.

2.2. LGD and EAD Correlations

The relationship between LGD and EAD is more complex. Roesch and Scheule (2010) estimated a positive correlation of 0.2 for retail loans, while Jacobs Jr and Karagozoglu (2011) found a negative correlation of -0.3 for corporate loans. The direction of the LGD-EAD correlation likely depends on the type of credit exposure. For revolving credit such as credit cards, a positive correlation may arise because higher utilisation leads to both higher EAD and lower recovery rates. For term loans, a negative correlation is more plausible, as higher EAD may be associated with earlier default and less time for losses to accrue.

2.3. PD and LGD Correlations

Several studies have found significant positive correlations between PD and LGD. Altman, Brady, Resti, and Sironi (2005) analysed a large sample of corporate bonds and loans, estimating the PD-LGD correlation to be ≈ 0.3. Qi and Zhao (2011) used a Bayesian model to estimate a correlation of 0.5 for a portfolio of retail loans. Leow and Mues (2012) found a correlation of 0.4 for UK residential mortgages.

Miu and Ozdemir (2006) explored the LGD rating philosophy concerning the downturn LGD requirement, demonstrating how conservatism in LGD to address the lack of correlations can be integrated into an acyclical Point-in-Time (PIT) framework. They also proposed a stylised model to comprehensively analyse and model the correlations between PD and LGD, leveraging historical default data from a loan portfolio, the correlations of LGD risk drivers among different obligors, and the correlation of systematic PD and LGD risk factors. For instance, even with moderate levels of idiosyncratic correlations and systematic correlations estimations, it is found that a substantial increase in mean LGD is necessary to achieve correct economic capital levels.

The implications of adopting different LGD philosophies, such as through-the-cycle (TTC) and PiT approaches, can also be considerable. While the PiT framework is often viewed as more responsive to current economic conditions, it may not adequately account for the cyclical nature of credit risk. Conversely, the TTC approach tends to smooth out fluctuations and may overlook immediate risks posed by rapidly changing economic conditions (Chawla, Forest Jr, & Aguais, 2017).

Common factors driving both PD and LGD, such as macroeconomic conditions, can explain the positive correlation. When the economy is weak, default rates tend to be higher, and recovery rates on defaulted loans tend to be lower.

2.4. PD, LGD and EAD Correlations

Financial institutions compute ECL for IFRS 9 by aggregating the products of PD, LGD, and EAD across their credit portfolios, but this simplified method assumes independence among PD, LGD, and EAD variables. When these are correlated, the approach underestimates ECL. Previous studies have shown that underestimation, which happens when the combined convexity and correlation effects are not taken into account, is around 20% to 30%. The convexity effect arises from the nonlinear relationship between PD and the underlying systematic factor, while the correlation effect pertains to the interrelations among PD, LGD, and EAD (Chawla et al., 2017).

Crook and Bellotti (2010) researched diverse classes of modelling techniques capable of integrating macroeconomic data. These classes encompass a broad classification into portfolio-level and loan-level models, and within the domain of loan-level models, they explore survival models, panel models, and correction factor models, whereas within portfolio-level models, they analyse loss distributions, Merton-type models, and econometric models. In their study, Crook and Bellotti (2010) explored time-varying and dynamic models for default risk in consumer loans and examined how variables impact credit risk, using dynamic time-varying models that consider correlations between credit risk variables (PD, LGD, EAD) and how these change over time.

The methodology employed by Miu and Ozdemir (2023) mirrors that of their earlier study, where PD and LGD are influenced by a single systematic factor (Miu & Ozdemir, 2006). The stylised model captures the various correlations between PD, LGD, and EAD, and Miu and Ozdemir (2023) argue that this framework provides a more accurate approach for estimating the required ECL. Using simulated scenarios based on historical data, the mean LGD was found to require significant adjustment (35%-41% for corporate portfolios and $\approx 16\%$ for mid-market portfolios) to compensate for the lack of correlation modelling. Miu and Ozdemir (2023) economic model enhances previous research by incorporating a stochastic EAD factor that correlates with both PD and LGD. Neglecting the convexity effect of the PD function results in a 20% underestimation of ECL, and disregarding the correlation effects among PD, LGD, and EAD leads to an extra 20% underestimation. This combined underestimation of 40% exceeds a bank's tolerance level when modelling its IFRS-9 ECL.

The model accounts for normalised PD and LGD risks of individual obligors, segmenting them into systematic and idiosyncratic components. Assuming that loan facilities are lines of credit that allow borrowers to make additional draws up to a set limit, EAD has two parts: 'the "used" amount (the amount that is currently being drawn) and the "unused" amount (the amount that is available minus the amount that is currently being drawn). As credit conditions deteriorate, obligors utilise a larger portion of the facility through further drawdowns to meet working capital needs and other obligations. By extending existing frameworks to account for these components of EAD risk, the full correlation structure among PD, LGD, and EAD risks is captured.

3. Materials and Methods

3.1. Methodology

To model the EAD, suppose a loan facility is a line of credit where the obligor is permitted further drawdowns, including the amount already drawn, up to a predetermined limit. EAD thus comprises the currently drawn amount (utilised) and the difference between the available limit and the currently drawn amount (unutilised).

In deteriorating credit conditions, obligors use larger facility portions, using more drawdowns to meet, e.g., working capital requirements or other obligations. This additional utilisation rate is typically accounted for using a credit conversion factor (CCF for which $0 \leq CCF \leq 1$). It is a BCBS requirement that $EAD_t^i \geq Drawn_t^i$ for obligor i at time t. For obligor i, then, EAD is

$$EAD_t^i = Drawn_t^i + CCF_t^i \cdot \left(Limit_t^i - Drawn_t^i\right) \quad (1)$$

Where CCF_t^i is a stochastic variable and $Limit_t^i$ and $Drawn_t^i$ are known constants at time t. For these reasons, CCF is modelled for EAD, rather than EAD itself.

Building upon Miu and Ozdemir (2006) this study considers a single systematic risk driver, X_t (assumed to be standard normally distributed, i.e., $X_t \sim N(0,1)$), which influences variations in PD, LGD, and CCF risks to varying degrees. Market-wide, systematic PD, LGD and CCF risks at time t (P_t, L_t and CCF_t) are influenced by X_t using (2) through (4):

$$P_t = \beta_{PD} \cdot X_t + \varepsilon_{PD,t} \quad (2)$$

$$L_t = \beta_{LGD} \cdot X_t + \varepsilon_{LGD,t} \quad (3)$$

$$C_t = \beta_{CCF} \cdot X_t + \varepsilon_{CCF,t} \quad (4)$$

The β coefficients govern the degree of impact on the systematic risks. Independent of X_t and assumed to be mutually independent are the residual changes ($\varepsilon_{PD,t}$, $\varepsilon_{LGD,t}$ and $\varepsilon_{CCF,t}$) – also assumed to be normally distributed with variances such that the systematic risks P_t, L_t and CCF_t are also normally distributed.

Table 1 displays cross correlations between systematic risks using equations (2) through (4).

Table 1. Systematic risk cross correlations.

Correlation between		Calculation	Denoted by
PD and LGD	P_t and L_t	$\beta_{PD} \cdot \beta_{LGD}$	R^2_{G1}
LGD and CCF	L_t and C_t	$\beta_{LGD} \cdot \beta_{CCF}$	R^2_{G2}
PD and CCF	P_t and C_t	$\beta_{PD} \cdot \beta_{CCF}$	R^2_{G3}

Linkages and derivations of systematic cross correlations are shown in Figure 1.

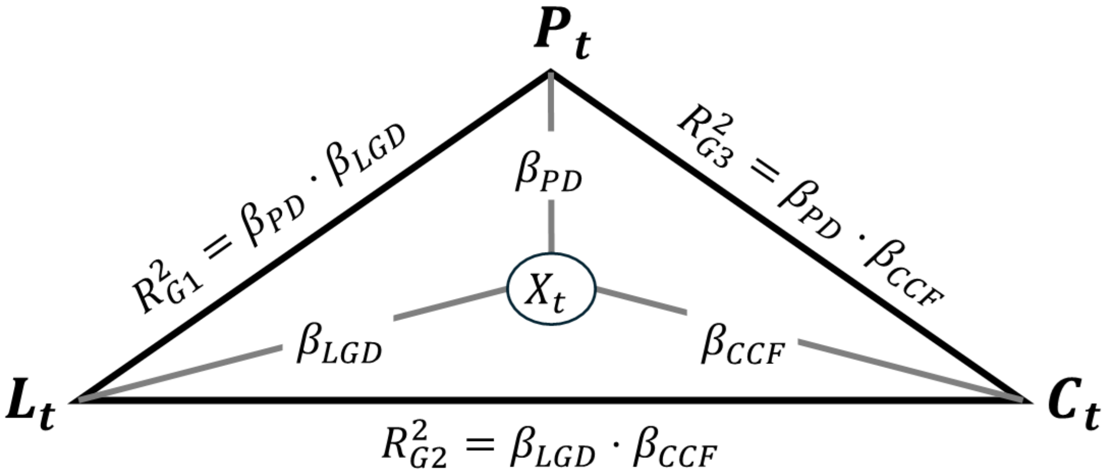

Figure 1. Schematic of systematic risk cross correlations.

In such a stylised economy, it is assumed that borrowers credit risks are uniform and that individual PD risk (p_t) is affected by both systematic PD risk (P_t) and (also normally distributed) obligor-specific PD risk ($e_{PD,t}$). For obligor i

$$p_t^i = R_{PD} \cdot P_t + \sqrt{1 - R_{PD}^2} \cdot e_{PD,t}^i \quad (5)$$

Where R_{PD}, the sensitivity of individual risks to systematic PD risk is assumed to be uniform across all obligors. Individual obligor PD risks are governed by the random variable p_t a normalised function of the inverse of the borrower's asset value under the Merton framework. As an example, for an obligor PD of 5%, the corresponding default point (DP) $= N^{-1}(5\%) = -1.645$, so the obligor will default if the realised borrower-specific $p_t > -DP$.

The structure of (2) through (4) indicates that all cross-correlations will be ≥ 0 if the β coefficients are all ≥ 0. In deteriorating market conditions, X_t increases, P_t increases (so PD increases) and L_t and C_t also increase (meaning LGD and EaD increase). The systematic risk factor introduces pairwise correlation in PD risks among obligors – given by R^2_{PD}. The larger these pair-wise correlations, the more likely are joint defaults among multiple obligors.

The obligors individual LGD risk, l_t may also be decomposed into systematic and idiosyncratic components such that the realised LGDs of two defaulted facilities are simultaneously governed by the same systematic factor L_t and their own individual borrower-specific factors. For obligor i

$$l_t^i = R_{LGD} \cdot L_t + \sqrt{1 - R_{LGD}^2} \cdot e_{LGD,t}^i \quad (6)$$

Where R_{LGD}, the sensitivity of individual risks to the systematic LGD risk, is again uniform across obligors and $e_{PD,t}$ is obligor-specific, normally distributed LGD risk. Once an obligor has defaulted, realised LGDs are assumed to follow a β distribution with shape parameters a_{LGD} and b_{LGD}, i.e.,

$$LGD_t^i = B^{-1}\left(\Phi\left(l_t^i\right); a_{LGD}, b_{LGD}\right) \quad (7)$$

Values of $a_{LGD} = 0.47$ and $b_{LGD} = 0.80$ result in an LGD distribution with a mean of 37% and standard deviation of 32%. The realised LGD value is governed by the random variable l_t (6) which is standard normally distributed and a transformed distribution of the empirical LGD's β distribution. The higher l_t, the higher the obligor's LGD. The pairwise correlation in LGD risks among obligors, R^2_{LGD}, arises from the systematic risk factor X_t. The larger pair-wise correlations, the more likely it is that higher (or lower) than average LGDs will be realised for two defaulted facilities simultaneously. For obligor i,

$$c_t^i = R_{CCF} \cdot C_t + \sqrt{1 - R_{CCF}^2} \cdot e_{CCF,t}^i \quad (8)$$

Having defaulted, an obligor's realised CCF is assumed to follow a β distribution with shape parameters a_{CCF} and b_{CCF}, i.e.,

$$CCF_t^i = B^{-1}\left(\Phi\left(c_t^i\right); a_{CCF}, b_{CCF}\right) \quad (9)$$

Values of $a_{CCF} = 1.14$ and $b_{CCF} = 1.70$ result in a β-distributed CCF with a mean of 40% and a standard deviation of 25%. The variable c_t follows a standard normally distributed and is a transformed distribution of the CCF's empirical β distribution. A one-to-one monotonic mapping is realised between c_t and CCF – bounded by 0 and 1 and the higher c_t, the higher is the obligor's CCF. R_{CCF}, the sensitivity of individual risks to systematic CCF risk, is uniform across obligors. R_{CCF}^2 is the pair-wise correlation in CCF risks among obligors due to X_t so the larger pair-wise correlations, the more likely that higher (or lower) than average CCFs will be realised for two defaulted facilities simultaneously. Having determined the CCF value, the corresponding EAD can then be calculated using (1). Note that if the loan exposure is fixed (i.e., no additional credit is available and the credit line has been fully utilised, so $Limit_t^i = Drawn_t^i$) the second term of (1) is $CCF_t^i \cdot \left(Limit_t^i - Drawn_t^i\right) = 0$.

Cross correlations between PD/LGD, LGD/CCF, and PD/CCF for the same obligor arise from both systematic risk factors and obligor-specific risk factors. Obligor-specific events (which, by definition, might be independent of market-wide systematic risks) can increase any of the obligor's PD, LGD, and CCF. Obligor-specific impacts are modelled by introducing correlations between borrower-specific residuals $e_{PD,t}$, $e_{LGD,t}$ and $e_{CCF,t}$ in (5) through (7) via an obligor-specific factor model in which x_t represents the borrower-specific credit risk factor (assumed to be standard normally distributed). For obligor i, then:

$$e_{PD,t}^i = \theta_{PD}^i \cdot x_t^i + \varepsilon_{PD,t}^i \quad (10)$$

$$e_{LGD,t}^i = \theta_{LGD}^i \cdot x_t^i + \varepsilon_{LGD,t}^i \quad (11)$$

$$e_{CCF,t}^i = \theta_{CCF}^i \cdot x_t^i + \varepsilon_{CCF,t}^i \quad (12)$$

Coefficients θ_{PD}^i, θ_{LGD}^i and θ_{CCF}^i govern the impact severity of x_t on $e_{PD,t}$, $e_{LGD,t}$ and $e_{CCF,t}$. Residual changes ($\varepsilon_{PD,t}^i$, $\varepsilon_{LGD,t}^i$ and $\varepsilon_{CCF,t}^i$) are independent of x_t^i and one another and normally distributed with variances such that $e_{PD,t}^i$, $e_{LGD,t}^i$ and $e_{CCF,t}^i$ are standard normally distributed.

Cross correlations between idiosyncratic risks, determined from (10) through (12), are shown in Table 2.

Table 2. Idiosyncratic risk cross correlations.

Correlation between	Calculation	Denoted by
$e_{PD,t}^i$ and $e_{LGD,t}^i$	$\theta_{PD}^i \cdot \theta_{LGD}^i$	R_{S1}^2
$e_{LGD,t}^i$ and $e_{CCF,t}^i$	$\theta_{LGD}^i \cdot \theta_{CCF}^i$	R_{S2}^2
$e_{PD,t}^i$ and $e_{CCF,t}^i$	$\theta_{PD}^i \cdot \theta_{CCF}^i$	R_{S3}^2

Linkages and derivations of idiosyncratic cross correlations are shown in Figure 2.

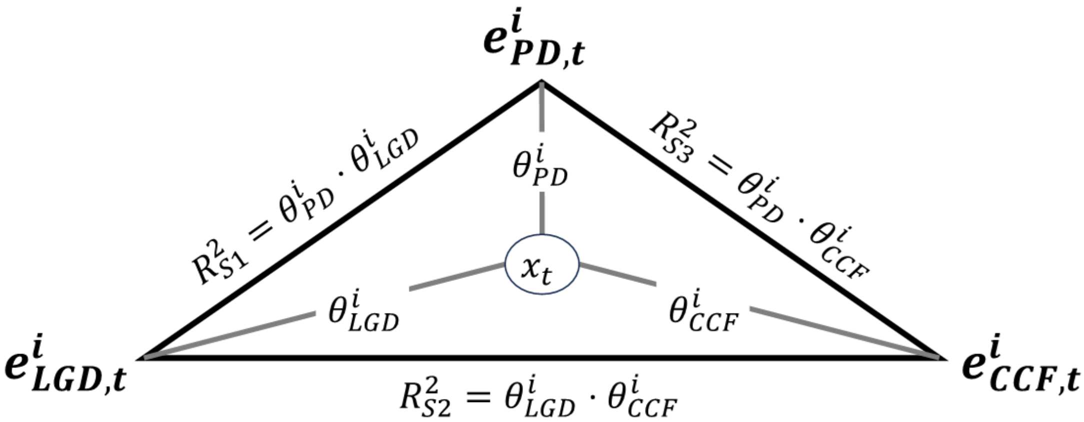

Figure 2. Schematic of systematic risk cross correlations.

The overall situation described above is shown schematically in Figure 3.

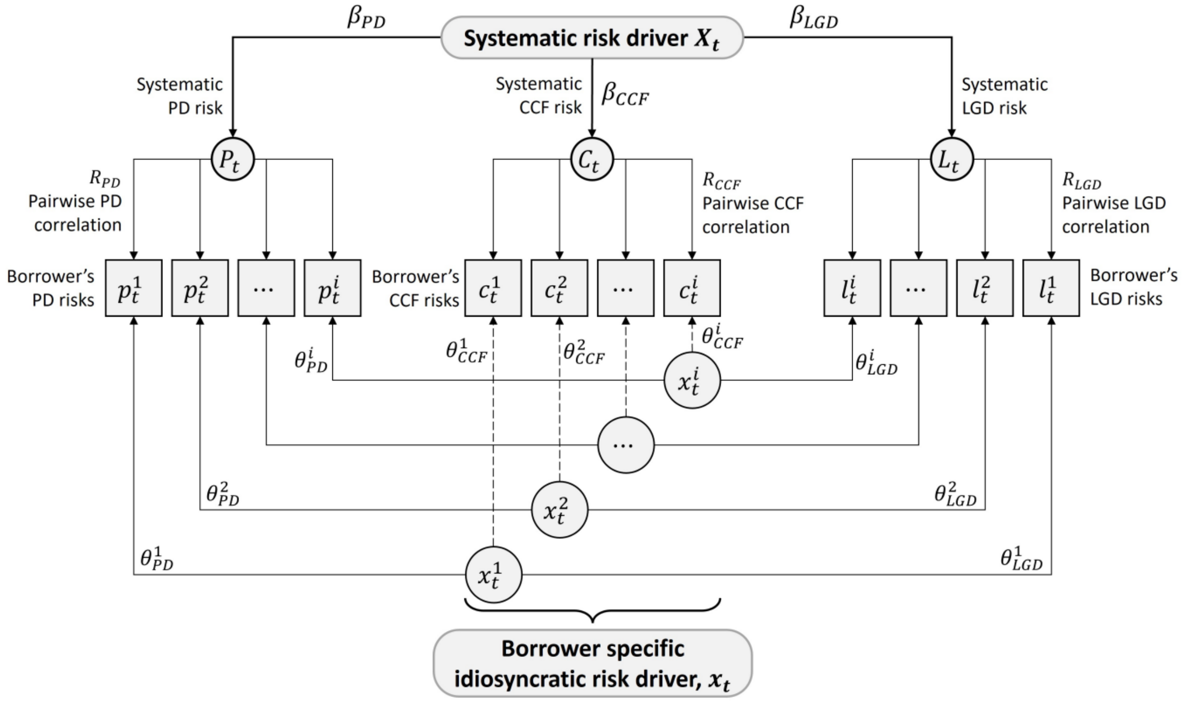

Figure 3. Links between systematic and idiosyncratic risk drivers and PD, LGD, and EAD.

3.2. Data

Simulated data were used. The parameters were varied, in turn, between 0.00 (or ≈ 0) and 1.00 (or ≈ 1) while holding the others constant to isolate individual impacts of parameters variation on credit losses-see Table 3.

Table 3. Parameter variation, range, impact and linkage equations. Other variables used were: v is a uniform random variable such that $0 \leq v \leq 1$, limit ($L = 100$), drawn ($D = 40$), long run PD ($PD_{LR} = 5\%$).

Variable	Range	Affects	Via
$\beta_{PD}, \beta_{LGD}, \beta_{CCF}$	$0 \leq \beta \leq 1$	$R_{G1}^2, R_{G2}^2, R_{G3}^2$	$R_G^2 = \beta_1 \cdot \beta_2$
$\theta_{PD}, \theta_{LGD}, \theta_{CCF}$	$0 \leq \theta \leq 1$	$R_{S1}^2, R_{S2}^2, R_{S3}^2$	$R_S^2 = \theta_1 \cdot \theta_2$
R_{PD}, R_{LGD}, R_{CCF}	$0 \leq R \leq 1$	p_t^i, l_t^i, c_t^i	$R \cdot P + \sqrt{1 - R^2} \cdot e_{PD}^i$ (etc)
$\varepsilon_{PD}, \varepsilon_{LGD}, \varepsilon_{CCF}$	$N^{-1}(0 \leq v \leq 1)$	P_t, L_t, C_t	$\beta \cdot X + \varepsilon$
$\varepsilon_{PD}^i, \varepsilon_{LGD}^i, \varepsilon_{CCF}^i$	$N^{-1}(0 \leq v \leq 1)$	$e_{PD}^i, e_{LGD}^i, e_{CCF}^i$	$\theta \cdot x + \varepsilon^i$
X (Systematic)	$N^{-1}(0 \leq v \leq 1)$		
x_t^i (Idiosyncratic)	$N^{-1}(0 \leq v \leq 1)$		
PD	$p_t^i > N^{-1}(1 - PD_{LR})$		
LGD	$\mathrm{B}^{-1}(N^{-1}(l_t^i), 10.69, 13.06)$	Figure 4(a)	
CCF	$\mathrm{B}^{-1}(N^{-1}(c_t^i), 0.93, 0.40)$	Figure 4(b)	
EAD	$L + CCF \cdot (L - D)$	Figure 4(d)	
Credit loss	Default: $LGD \cdot EAD$ No default: 0	Figure 4(c)	

Figures 4(a) through (d) show simulation results summary.

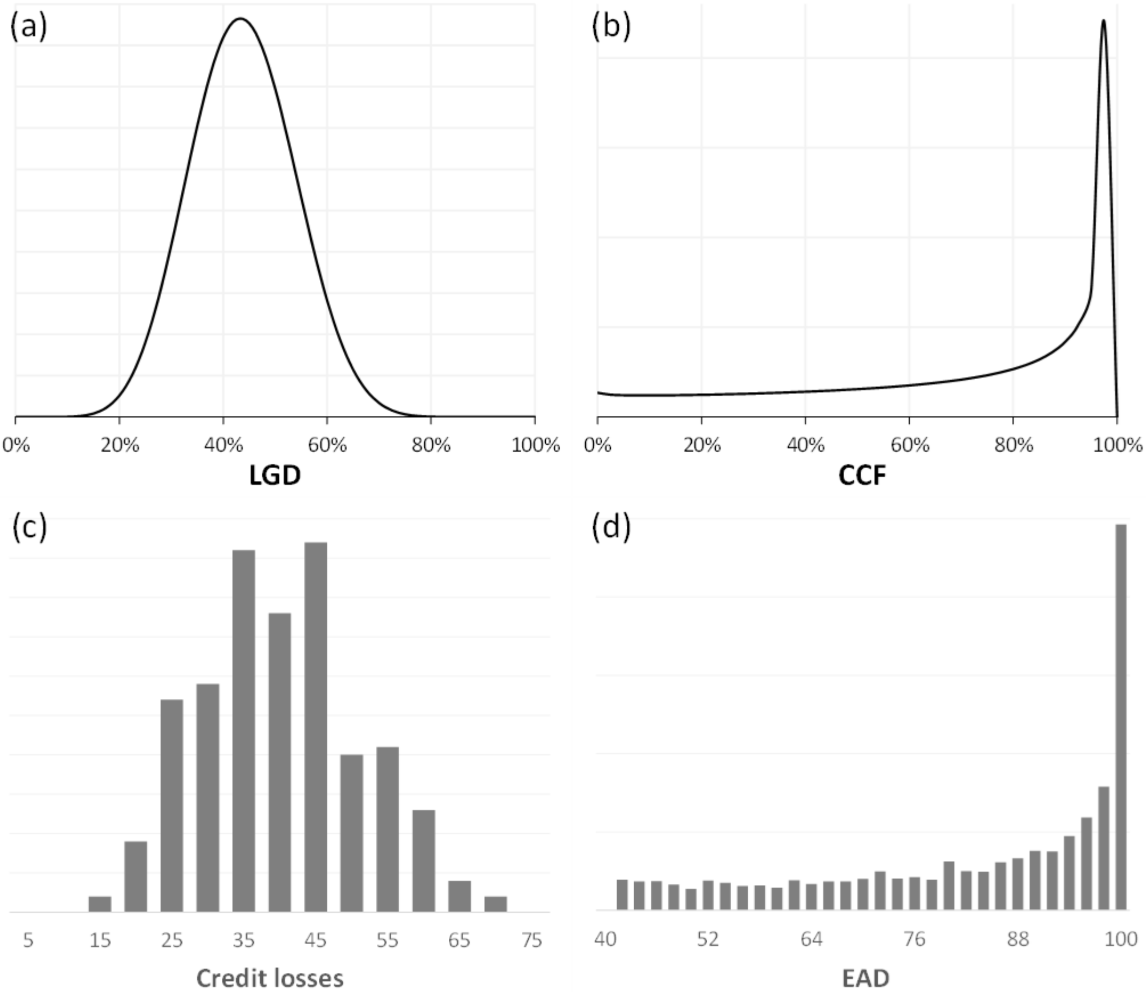

Figure 4. Results of simulations. Probability density of **(a)** LGD, **(b)** CCF, **(c)** Credit loss and **(d)** EAD.

4. Results

The pairwise correlations among PD, LGD, and EaD significantly influence the estimation of ECL. When PD, LGD, and EaD are modelled independently, the ECL is calculated as the sum of the product of these three parameters across a portfolio of loans. However, when the correlations are factored in, the ECL increases due to the compounding effect of these risk factors during economic downturns. Researchers have found that if you don't take into account the convexity of the PD function and the correlations between PD, LGD, and EaD, you might get the wrong estimate of ECL. This is because of the effects of convexity and correlations. This significant underestimation implies that institutions may hold insufficient capital to cover potential losses under IFRS-9 and have ineffective capital allocation when advantageous economic conditions arise.

To illustrate the impact of these correlations on ECL, we provide a simulated numerical example using simulated credit losses for a portfolio under various scenarios (5a -f). These simulations capture the joint distribution of PD, LGD, and EaD. The results show substantial effects on ECL when correlations are considered.

Understanding the impact of these correlations is crucial for financial institutions for several reasons. Firstly, it affects capital adequacy and regulatory compliance. Accurate modelling of these correlations is essential for calculating regulatory capital requirements under IFRS-9 standard. By capturing the full correlation structure, institutions can avoid underestimating capital needs, especially in stress scenarios. Secondly, the correlations between PD, LGD, and EaD have significant implications for liquidity management. During downturns, higher EaD (due to increased drawdowns on credit lines) coupled with higher LGD can strain an institution's liquidity position. The framework helps in preparing for such scenarios by better informing funding strategies.

Given scenario and shown by Figure 5a: β low, θ high and $0 \leq R \leq 1$ and substituting (2) into (5) gives

$$p_t^i = R_{PD} \cdot (\beta_{PD} \cdot X_t + \varepsilon_{PD,t}) + \sqrt{1 - R_{PD}^2} \cdot e_{PD,t}^i \qquad (13)$$

Then, as $R_{PD} \to 0$, $p_t^i \to e_{PD,t}^i$ and as $R_{PD} \to 1$, $p_t^i \to \beta_{PD} \cdot X_t + \varepsilon_{PD,t}$ but if $\beta_{PD} \approx 0$ then as $R_{PD} \to 1, p_t^i \to \varepsilon_{PD,t}$. Since $e_{PD,t}^i$ values are correlated by θ, then as $R_{PD} \to 0$ obligors become more correlated about $e_{PD,t}^i$. More correlated losses result in higher losses, while an increase in R_{PD} or while $R_{PD} \to 1$, obligors become less correlated (as there is less dependence on a θ influenced variable). This idea propagates through PD, LGD, and CCF estimations in a similar way, as the same mathematics holds true. This demonstrates the substantial impact that including these correlations has on credit loss estimations and highlights the compounded risk during economic downturns.

Substituting (10) into (13) gives:

$$p_t^i = R_{PD} \cdot \left(\beta_{PD} \cdot X_t + \varepsilon_{PD,t}\right) + \sqrt{1 - R_{PD}^2} \cdot \left(\theta_{PD}^i \cdot x_t^i + \varepsilon_{PD,t}^i\right) \qquad (14)$$

Table 4 presents these influences on p_t^i:

Table 4. Summary of simulation results and impact of parameter variation.

Simulation condition	Approaching 0 simulation	Approaching 1 simulation
$0 \to R \to 1$	$\theta \approx 0$	$\theta \approx 1$
$\beta \approx 0$	$\varepsilon_{PD,t}^i \to \varepsilon_{PD,t}$	$x_t^i + \varepsilon_{PD,t}^i \to \varepsilon_{PD,t}$
$\beta \approx 1$	$\varepsilon_{PD,t}^i \to X_t + \varepsilon_{PD,t}$	$x_t^i + \varepsilon_{PD,t}^i \to X_t + \varepsilon_{PD,t}$
$0 \to \beta \to 1$	$\theta \approx 0$	$\theta \approx 1$
$R \approx 0$	$\varepsilon_{PD,t}^i$	$x_t^i + \varepsilon_{PD,t}^i$
$R \approx 1$	$\varepsilon_{PD,t} \to X_t + \varepsilon_{PD,t}$	$\varepsilon_{PD,t} \to X_t + \varepsilon_{PD,t}$
$0 \to \theta \to 1$	$R \approx 0$	$R \approx 1$
$\beta \approx 0$	$\varepsilon_{PD,t}^i \to x_t^i + \varepsilon_{PD,t}^i$	$\varepsilon_{PD,t}$
$\beta \approx 1$	$\varepsilon_{PD,t}^i \to x_t^i + \varepsilon_{PD,t}^i$	$X_t + \varepsilon_{PD,t}$
$0 \to R \to 1$	$\theta \approx 0$	$\theta \approx 1$
$\beta \approx 0$	$\varepsilon_{PD,t}^i \to \varepsilon_{PD,t}$	$x_t^i + \varepsilon_{PD,t}^i \to \varepsilon_{PD,t}$
$\beta \approx 1$	$\varepsilon_{PD,t}^i \to X_t + \varepsilon_{PD,t}$	$x_t^i + \varepsilon_{PD,t}^i \to X_t + \varepsilon_{PD,t}$

Figure 5a: $\beta \downarrow$ - low systematic risk, low correlation with market factors, $\theta \uparrow$ - high correlation between borrower-specific risk drivers. As R (pair wise correlations between obligors) increase. Figure 5b presents the inverse relationship between β and θ as R increases.

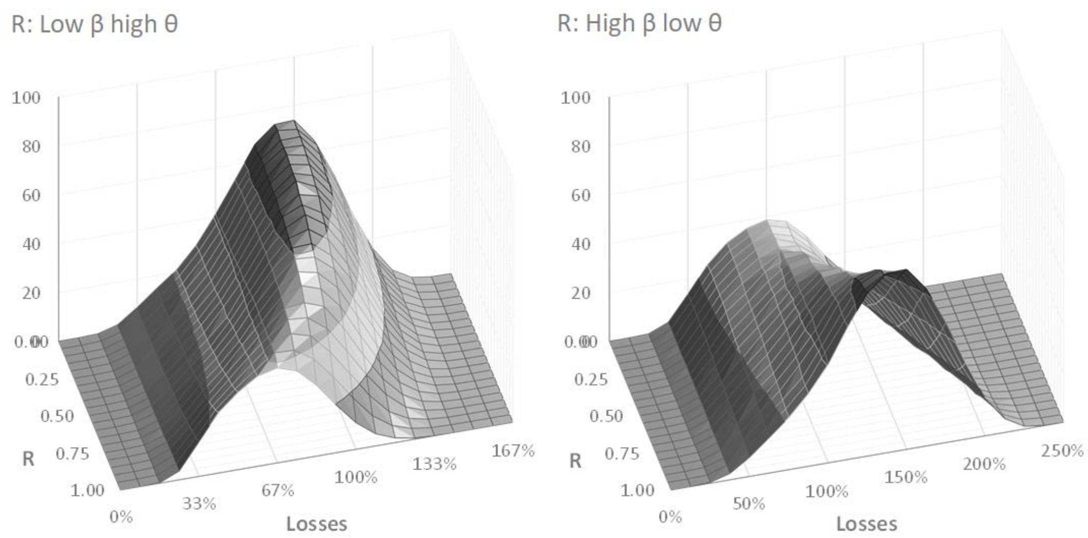

Figure 5(a). Low β, high θ and (b) high β, low θ.

Figure 5c: $\beta \uparrow$ - high systematic risk, high correlation with market factors, $\theta \uparrow$ - high correlation between borrower-specific risk drivers. As R (pairwise correlations between obligors) increase. Figure 5d presents the inverse effect between β and θ as R increases.

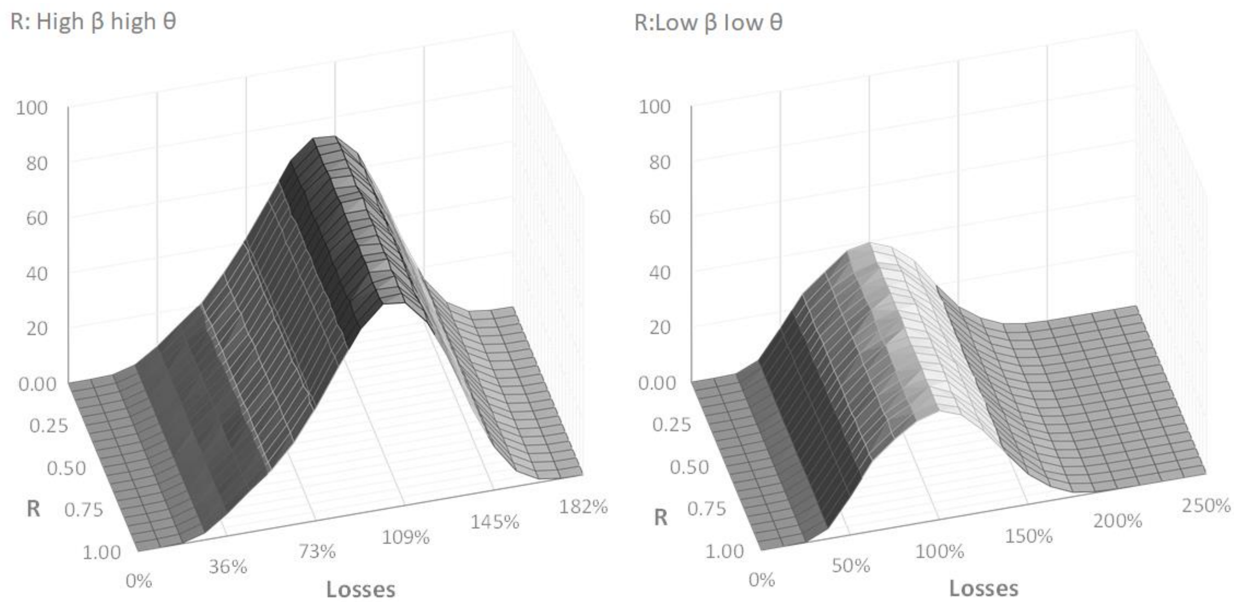

Figure 5(c). High β, high θ and (d) low β, low θ.

Figure 5e: $\beta \downarrow$ - low systematic risk, low correlation with market factors, $R \uparrow$ - pairwise correlations between obligors. As θ (correlation borrower-specific risk drivers) increase. Figure 5d presents the inverse effect between β and R as θ increases.

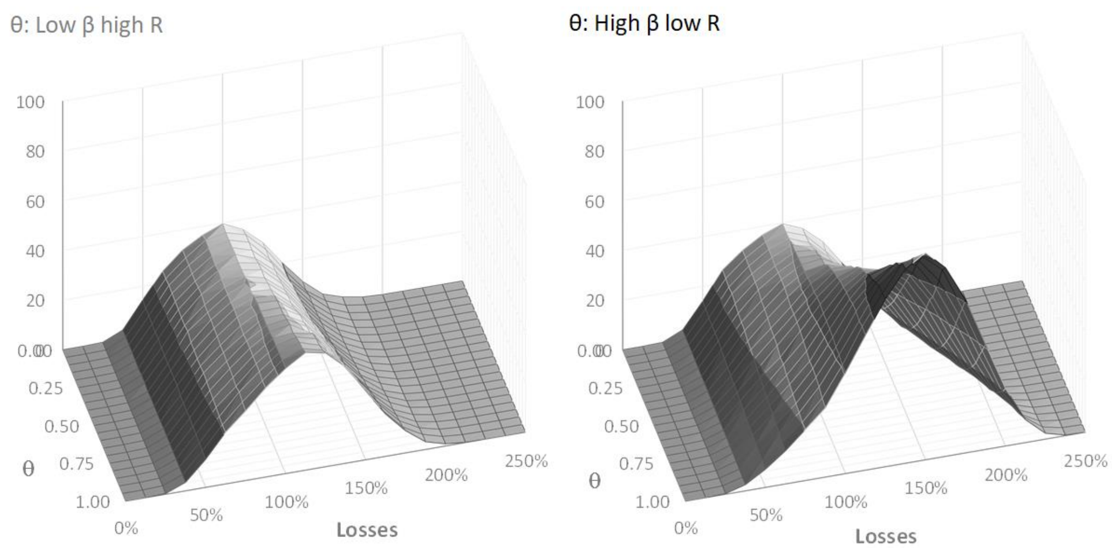

Figure 5(e). Low β, high R and (f) high β, low R.

Figure 5g: $\theta \downarrow$ - low correlation borrower-specific risk drivers, low correlation with market factors, $R \uparrow$ high pairwise correlations between obligors. As β (systematic risk) increases. Figure 5h presents the inverse effect between β and R as θ increases.

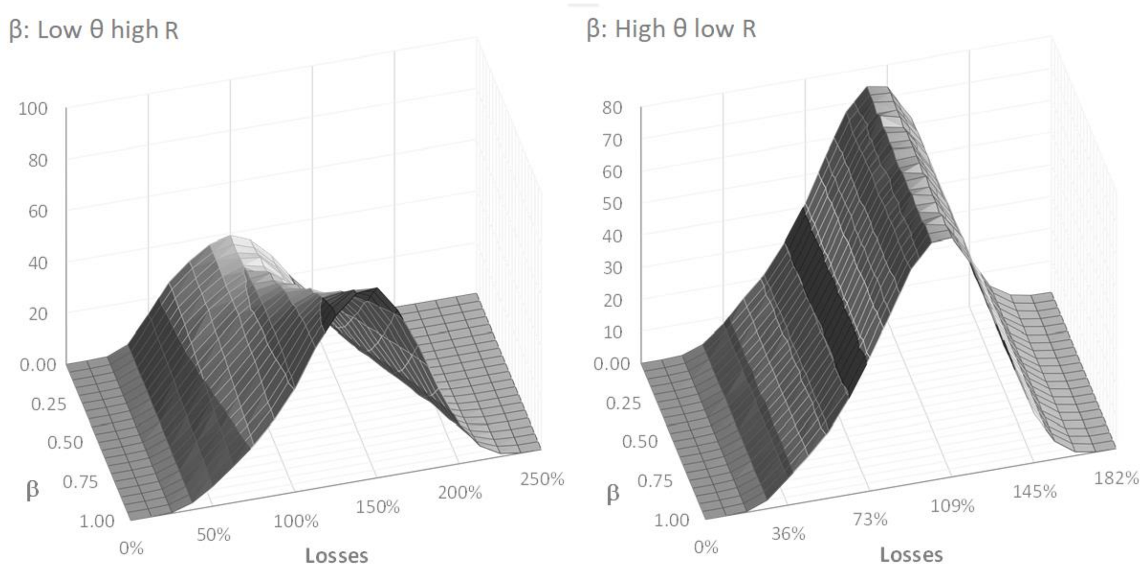

Figure 5(g). Low θ, high R and (h) high θ, low R.

Figure 5i: $\theta \downarrow$ - high correlation between borrower-specific risk drivers, high correlation with market factors, $R \uparrow$ high pairwise correlations between obligors. As β (systematic risk) increases. Figure 5j presents the inverse effect between θ and R as β increases.

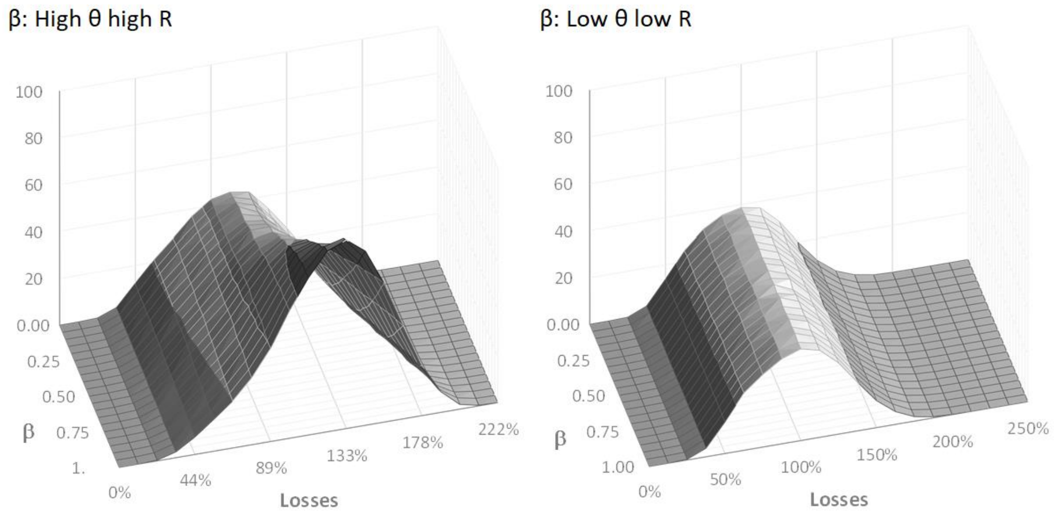

Figure 5(i). High θ, high R and (j) high θ, low R.

Figure 5k: $\beta \uparrow$ - high systematic risk, high correlation with market factors, $R \uparrow$ - pairwise correlations between obligors. As θ (correlation borrower-specific risk drivers) increase. Figure 5l presents the inverse effect between β and R as θ increases.

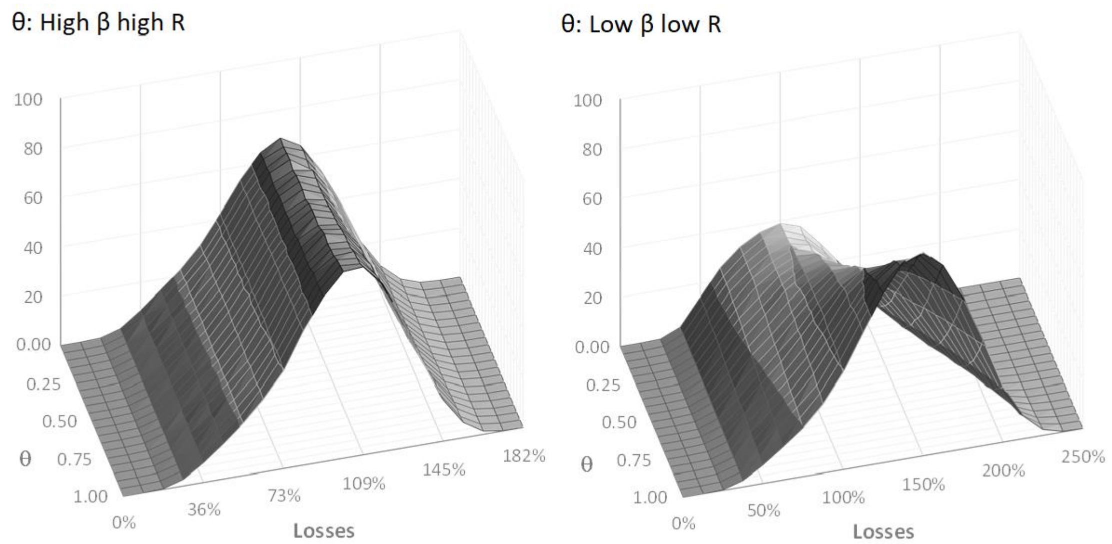

Figure 5(k). High β, high R and (l) low β, low R.

5. Conclusion

This paper extends the approach provided by Miu and Ozdemir (2023) for a more advanced economic framework that captures the correlations between PD, LGD, and EaD to enhance the accuracy of ECL estimation calculations for institutions. This framework gives a more accurate way to estimate ECL according to IFRS-9 rules by taking into account that EaD is a random variable and the correlations between PD, LGD, and EaD. The framework addresses the significant impact of these correlations on risk assessment and capital requirements.

The paper also highlights the importance of accounting for the correlations among PD, LGD, and EaD, which are often overlooked in traditional credit risk models. Three types of correlations are identified and investigated: PD-LGD, PD-EaD, and LGD-EaD. The PD-LGD correlation has been studied the most because it shows how the chance of default and the chance of recovery tend to move in sync with each other during economic cycles. The PD-EaD correlation highlights the dependency between the likelihood of default and the level of exposure at default, especially when borrowers increase their drawdowns in anticipation of potential default. The LGD-EaD correlation shows how recovery rates are related to exposure levels. It suggests that the recovery rate (LGD) can change as potential exposure (EaD) rises, especially when the economy is bad. The proposed approach integrates these correlations to improve the accuracy of capital requirements under IFRS9 guidelines.

The framework assumes uniform sensitivity parameters (β) and correlation structures across different obligors and portfolios, which may not hold true in all cases. Future models should consider differentiating these parameters based on obligor types, loan characteristics, and economic conditions. Another key assumption is the use of a single systematic risk factor; however, incorporating multiple risk drivers could provide a more granular understanding of the joint behaviours of PD, LGD, and EaD. Another suggestion is that financial institutions should implement rigorous back-testing procedures and stress tests that account for the joint distribution of these risk parameters, ensuring that their capital and liquidity buffers are sufficient to withstand extreme but plausible adverse scenarios.

Future research could focus on refining the estimation techniques for PD, LGD, and EaD correlations, especially in the context of different economic cycles and across varied credit products. Exploring non-linear dependencies and incorporating machine learning techniques to dynamically adjust correlations based on market conditions could further enhance the robustness of credit risk models. Additionally, more empirical studies using real-world data across different jurisdictions and regulatory environments would help validate and calibrate the proposed framework, ensuring its adaptability and accuracy in diverse financial landscapes.

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