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A Markov-Switching model of public debt in South Africa

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Abstract

This research aimed to explore the sustainability of public debt with changes in the South African regime from 1960 to 2020. Annual data was obtained from the South Africa Reserve Bank, and the Markov-switching autoregressive (MS-AR) model was employed to describe the public debt process in South Africa. Compared to traditional endogenous modelling, the process explicitly allows for the possibility of regime changes. Regime 1 defines the stages of high public debt, and regime 2 describes periods of a downward trend in public debt. The results reveal that in regime 1, there is a 98% likelihood of remaining in high public debt and a lower probability of 1.76% switching to a lower public debt regime. Also, when the system is in a lower debt regime, there is a 98.2% likelihood of remaining in a lower public debt state and a lower probability of 1.74% switching to a higher public debt state. The expected average duration of a period of higher public debt is 56 years, while the average duration of a lower public debt regime is 57 years. This implies that it is only in an extreme event that public debt can switch from a high-debt regime to a low-debt regime and vice versa. Hence, it is suggested that the government implement policies knowing that public debt will not decline suddenly.

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1. Introduction

Debt has been increasing across many countries worldwide, and most governments have been struggling with fiscal discipline to curb the debt. Governments finance their deficits through borrowing from both domestic and external financial institutions. Borrowing comes with an interest payment requirement, which increases the debt burden. In the past decades, public debt levels in South Africa have continuously increased. From 1960 to 1979, the total gross loan debt as a % of Gross Domestic Product (GDP) fluctuated between 33.4% and 39.9 %. From 1980 to 1990, the total gross loan debt as a percentage of GDP was between 26% and 32%. From 1991 to 1999, debt increased from 28.3% to 42.8% (South African Reserve Bank, 2022).In 1999, the Public Finance Management Act was passed to reduce inherited high foreign public debt. It mentioned that the government should focus more on domestic financial sources to finance the budget deficit (Saungweme & Odhiambo, 2020). From 2000 to 2010, the total gross loan debt as a percentage of GDP, decreased from 40% and 28.3%. As of 2011, in the aftermath of the 2008/2009 sovereign debt crisis, South Africa experienced a continuous increase in the total gross loan debt as a percentage of GDP, from 31.9% in 2011 to 70.2% in 2021 (South African Reserve Bank, 2022). More public debt acquired in 2020 by the South African government was used to increase expenditures during the COVID-19 pandemic on the health sector, social welfare, and education (South African Reserve Bank, 2022). An increase in debt means an increase in debt service and a burden to the future generation. Most governments need to be cautious of their primary balance and be able to sustain the servicing costs of their debt, thereby avoiding default or major fiscal adjustments (Debrun, Ostry, Willems, & Wyplosz, 2019). This high debt rate has led to the need to assess the sustainability of debt in South Africa when the regime changes. Debt sustainability is defined by Djeutane and Munditi (2014) as a way of ensuring that government spending levels do not increase public interest payments on debt in the long term. According to Debrun et al. (2019), debt becomes unsustainable when debt cannot be serviced due to a large amount of debt. Naraidoo and Raputsoane (2015) mentioned that sustainability is linked with solvency, which is the ability of a country to service its debt in the long term. Authors such as Debrun et al. (2019) mentioned that if the primary balance cannot sustain higher debt interest rates, then the debt becomes explosive, hence the government would either default on its payments or undertake extra fiscal adjustment. The COVID-19 pandemic, which affected South Africa in 2020, made the debt level worse. Even thou the history of debt in South Africa has been sustainable, the rapid change in debt levels requires that debt be assessed. Some studies investigated the suitability of debt using different methods, such as Hakkio and Rush (1991); Curtaşu (2011), and Kaur, Mukherjee, and Ekka (2018), who used cointegration to examine debt sustainability. Kaur et al. (2018) and Bohn (1998) examine sustainability using debt and deficit, while Lankester Campos, Loaiza Marín, and Monge Badilla (2020) use the fiscal policy reaction function (FPRF). Stoian, Campeanu, and Roman (2007) used Ordinary Least Square whereas Beqiraj, Fedeli, and Forte (2018) used panel data to assess public debt sustainability. Baharumshah, Jibrilla, Sirag, Ali, and Muhammad (2016) used Enders and Siklos' threshold adjustment techniques and Granger causality tests, Ganyaupfu (2014) and Ganyaupfu and Robinson (2019) used the Vector Error Correction Model (VECM). The following authors made use of the Markov-switching model: Burger and Marinkov (2012); Olaoye and Olomola (2022), and Woldu (2022). Most of these studies used the multivariate technique to analyse debt sustainability. Hamilton (1989) and others used Markov switching to determine the different regimes, changing the mean and variance when debts are high and when debts are low, but theirs was a multivariable analysis. This study contributes by using a univariate technique called the Markov-Switching approachto see how long it will take for public debt to switch from one regime to another in South Africa.

The rest of this study is structured as follows: Section 2 provides the theoretical and empirical literature. Section 3 presents econometric methods and data sources. Section 4 provides the empirical results. Lastly, Section 5 outlines the conclusion of the study.

2. Literature Review

The continuous increase in public debt has triggered interest in examining whether the debt is sustainable. Debt sustainability is defined by Naraidoo and Raputsoane (2015) as being associated with solvency, which is the capability of a country to repair its debt in the long run. When a country is not able to raise adequate revenues in the long term to meet up with its debt commitments it is said to be insolvent. On the other hand, when the current debt and the present discounted value of the total expenditures do not surpass the present discounted value of the total revenues, then the country is said to be solvent. Debrun et al. (2019) iterate that when the primary balance cannot be enough for interest payments needed for the debt acquired, then the debt level will be likely to explode.

There are various ways in which sustainability could be assessed. It can be external, fiscal, or financial sector stability. External sustainability focuses on the current account deficit, assessing the appropriate exchange rate level, or making projections of the balance of payment with the associated debt. Financial sustainability focuses on how the financial institution's systems are stable in relation to public and external debt. On the other hand, fiscal sustainability can be assessed by using the public debt-to-GDP ratio and/or primary balance. Also, government expenditure and revenue ratios are also used to assess fiscal sustainability.

According to Domar (1944), for debt to be sustainable, the GDP share should decrease or remain constant in the medium and long term. For this condition to be kept in place, the interest rates on government loans should be lower than the growth rate of the country. For instance, the country's public debt should not rise faster than the GDP of the country. Bohn (1998) on the other hand, states that for a government budget to be sustainable, it will require effective government policies to work concurrently with the present value of the budget constraint. That is, the present value of expenditure by the government is equal to the present value of its revenues. This action is to ensure that there are no future budget deficits or increases in debt.

In South Africa, debt sustainability studies have been done using different techniques. Studies that used the fiscal reaction function are Burger, Stuart, Jooste, and Cuevas (2012). They used the fiscal reaction function to assess the sustainability of debt in South Africa, and the results revealed that South Africa had a sustainable fiscal policyfrom 1946 to 2008. Lankester Campos et al. (2020) used the fiscal policy reaction function (FPRF) and the government's intertemporal budget constraint to assess debt sustainability. The results show that public debt is unsustainable for the country of study. Burger and Marinkov (2012) examined the fiscal reaction function using the Markov-switching model for South Africa using data from 1972 to 2010. Their results confirmed fiscal sustainability in South Africa. Others used thethreshold adjustment techniques, such as Baharumshah et al. (2016), who,after examiningthe relationship between government revenue and government expenditure in South Africa, made use of Enders and Siklos' threshold adjustment techniques as well as the Granger causality tests from 1960 to 2013. The results revealed that there is fiscal debt sustainability in South Africa and recommended that fiscal policy instruments be maintained. Mahmood, Arby, and Sherazi (2014) compared debt sustainability in South Asian Association for Regional Cooperation (SAARC) countries (India, Sri Lanka, Pakistan, and Bangladesh) using the threshold debt ratios for assessing debt sustainability in those countries. The study results showed that the four countries have been experiencing fluctuations in the unsustainability of debt, mainly due to imbalances in the fiscal and current accounts. The Vector Error Correction Model (VECM) was used by the following authors: Ganyaupfu (2014) used the VECM to analyse fiscal sustainability in South Africa from 1990 to 2013; hence, the fiscal policy is sustainable. Ganyaupfu and Robinson (2019) used the Vector Error Correction Model from 1997 to 2016 to evaluate the sustainability of debt in South Africa, and the results confirm that debt is sustainable. Redda (2020) had contrary results when studying the sustainability of public debt and budget deficits in South Africa for the period 2000-2018. They employed VECM techniques as others did, but their study postulated that public debt in South Africa is unsustainable. The study elaborated that it is mainly due to insufficient tax revenue and the narrow budget deficit. The study further indicated that South Africa needs to increase its tax base to service its debt, and that can be achieved through job creation in the country, as that will increase economic growth.

Hakkio and Rush (1991); Curtaşu (2011); and Kaur et al. (2018) examined debt sustainability using the cointegration technique. To them, if cointegration exists between government revenue and government expenditure, then the fiscal imbalance can be managed in the future, hence sustainability is achieved. On the other hand, Kaur et al. (2018) and Bohn (1998) examine sustainability using debt and deficit variables. They state that sustainability is achieved when the relationship between the budget deficit-to-GDP ratio and the debt-to-GDP ratio is negative. If the relationship between deficit and debt is positive, then debt is not sustainable. Stoian et al. (2007) used Ordinary Least Square and got weak public debt sustainability. While Curtaşu (2011) examined sustainability using unit-root tests, cointegration tests, and fiscal reaction functions tests. Beqiraj et al. (2018) used panel data to assess public debt sustainability, and the study shows a long-run relationship between debt and structural primary balance; hence,debt is sustainable.

The Markov-Switching technique was used by different studies to analyse the sustainability of debt. Using the Markov-Switching approach, Olaoye and Olomola (2022) assessed whether public debt was sustainable in Sub-Saharan Africa. According to the findings, debt can be sustained in a situation where debt ratios are low, but not in one where they are high. Also, the transition between both regimes is highly persistent. Woldu (2022) used Markov-switching to assess fiscal sustainability in South Africa from 1960 to 2019. The results reveal that public debt was sustainable during the period. Also, when the regimes were considered, the No-Ponzi game condition was satisfied for the fiscal policy. Burger and Marinkov (2012) used Markov-switching estimations to assess fiscal sustainability for the period 1972Q1-2010Q4 in South Africa with primary balance and government debt. Regime 1 has fiscal pacifism and is pro-cyclical, while Regime 2 has fiscal activism and is counter-cyclical. Regime 1 is more persistent, while Regime 2 is identified over three brief periods. This implies that the government's fiscal policies are not sustainable. Most of these studies used the multivariate technique to analyse debt sustainability. This study will contribute by using a univariate technique. This study will fill the gap by using a univariate Markov-Switching approach to analyse the change in public debt in South Africa.

3. Econometric Methods and Data

3.1. Data Description

The data comprises yearly observations of the total loan debt of the national government measured in millions of rands taken from the South African Reserve Bank (SARB). Therefore, the study uses the abbreviation "DBTM" as the total loan debt of the national government. The sample runs from 1960 to 2020 and consists of 61 observations. The sample period of the study was chosen based on the availability of data to make enough observations.

3.2. Methodology

Econometricians and statisticians used conventional linear functions as fundamental frameworks for modelling. There are substantial indications that nonlinear modelling is at times appropriate, especially in analysing macroeconomic relationships that have changed in the regime. The Markov switching functions extend the simple exogenous probability framework by specifying a first-order Markov process for the regime probabilities. It starts with the description of the state likelihood specification and then provides the probability computation as well as the filtering and smoothing. To model the Markov Switching Autoregressive (MS-AR) process, this study precisely considers a two-regime switching model with a state-dependent variance and state-dependent mean for $DBTM = \left(\frac{x}{x_{t-1}}\right)$. Where $DBTM$ is defined as public debt and its growth rate is represented by x at time t.

$$x_t = c_{st} + \emptyset_1(x_{t-1} - c_{st-1}) + \emptyset_2(x_{t-2} - c_{st-2}) + e_t \quad (1)$$

$$e_t \sim i.i.d\ N(0, \delta_{st}^2)$$

Where: $c_{st} = c_0 S_{0t} + c_1 S_{1t} + c_2 S_{2t}$; $\delta_{st}^2 = \delta_1^2 S_{1t} + \delta_2^2 S_{2t} \quad (2)$

c_{st} is the regime-dependent mean, δ_{st}^2 is the regime-dependent variance, and the autoregressive parameters are $\emptyset_1$ and $\emptyset_2$ which are unique for different regimes. The intention is to model the regime S_t as the outcome of an unobserved 2-regime Markov chain with S_t independent of e_t for all$t$. Hamilton (1989) initiated the MS-AR model of two regimes which represents a model that switches regimes stochastically. Therefore, the Markov Switching 2 states with a process of order $p$of AR as follows:

$$x_t = c_1 + \alpha_{11}\gamma_{t-1} + \cdots \ldots\ldots + \alpha_{p1}\gamma_{t-p} + e_t \qquad S_t = 1 \qquad (3)$$
$$x_t = c_2 + \alpha_{12}\gamma_{t-1} + \cdots \ldots\ldots + \alpha_{p2}\gamma_{t-p} + e_t \qquad S_t = 2 \qquad (4)$$

Where the regime in function (x) are index by S_t. In these Equations 3 and 4, the parameters of the autoregressive part and the intercept are relying on the regime at time t. The regimes are presumed to be discrete unobservable variables. Therefore, regime 1 defines the stages of high public debt and regime 2 describes periods of a downward trend in public debt. The probability of being in a regime depends on the previous state, is presented in Equation 5:

$$P(s_t = j | s_{t-1} = i) = p_{ij}(t) \qquad (5)$$

Therefore, these probabilities are presumed to be time-invariant so that $p_{ij}(t) = p_{ij}$ for all, but this restriction is not required. This study Equation 6 present probabilities in a transition matrix.

$$p = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad (6)$$

Where the p_{11}+p_{12}=1 and p_{21}+p_{22}=1

Before the estimation of the MS-AR model, the study follows some preliminary analysis. Firstly, the paper tests the null hypothesis of linearity against the hypothesis of nonlinearity. The study makes use of the BDS test for linearity that Broock, Scheinkman, Dechert, and LeBaron (1996) invented. Secondly, use the Bierens unit root that Bierens (1997) developed to investigate the nonlinearity unit root. Thirdly, estimate the appropriately selected MS-AR model based on the Akaike Information Criterion (AIC) by Akaike (1974) and evaluate the model on diagnostic tests.

4. Empirical Results

4.1. Diagnostic Test Results

This start with the following diagnostic tests: graphical representation of public debt over the years, Descriptive statistics test, stability test and Bierens nonlinear unit root test results.

4.1.1. Total Public Debt for the Period 1960 - 2020

In Figure 1, the South African public debt measured in million Rands indicates a large, steady amount of debt from 1960 to the early 1990s. In 2008, public debt showed an upward trend from 2000 to 2020. Thus, the bar chart above indicates that public debt in South Africa may suggest a pattern of periods of high public debt and periods of lower public debt.

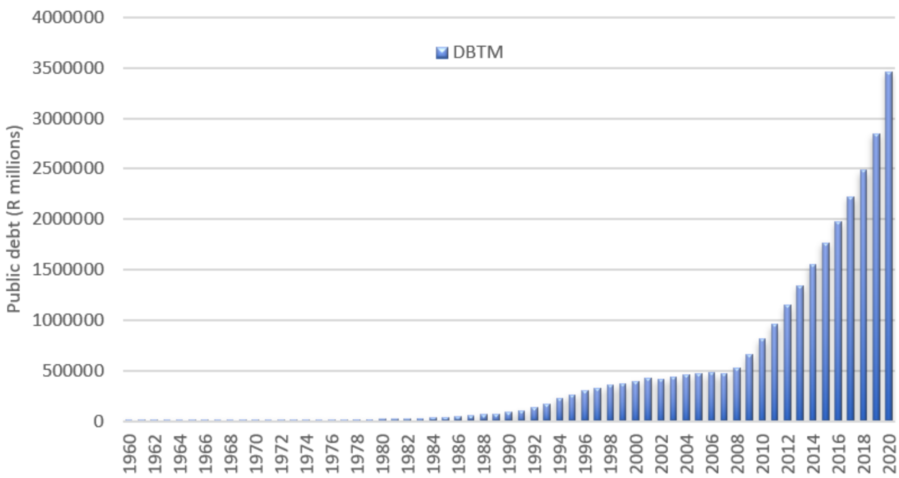


Figure 1. Total public debt for the period 1960 – 2020.

4.1.2. Descriptive Statistics Results

Table 1 provides the descriptive analysis results. From a statistical point of view, the purpose of descriptive statistics is to provide basic information about the variable(s) of interest in the study and to suggest the potential relationship between variables. It can be observed from the table that the minimum public debt measured in millions during the period was R2530, whereas the sample mean value was R461477.9 million and the maximum value was R3458234 million.

Table 1. Descriptive results of the study.

Descriptive measure	Value
Observations	61
Minimum	2530
Maximum	3458234
Sample mean	461477.9
Sample standard error	767062.18

4.1.3. BDS Test for Linearity Testing Results

Table 2 presents the BDS results, which specify that there is a nonlinearity specification on the South African public debt. All the p-values are less than 1%, thus suggesting a rejection of the null hypothesis that the variable is linearly formed. The findings in Table 1 imply that South African public debt is nonlinear and unstable, which is a signal of the unstable behaviour of economic time series data; hence, the variable can be modelled by fitting a nonlinear function.

Table 2.BDS test for linearity testing.

Dimension	BDS statistic	Std. error	z-statistic	Prob.
2	0.171	0.018	9.189	0.000
3	0.272	0.030	9.012	0.000
4	0.328	0.036	8.946	0.000
5	0.355	0.039	9.082	0.000
6	0.357	0.038	9.266	0.000

4.1.4. CUSUM of Squares Results

Also, the study applied the CUSUM of Squares test to detect any parameter or variance instability. Figure 2 is the demonstration of CUSUM of Squares and it shows that they are outside the boundary of the 5% significance level. This suggests instability and also that public debt is nonlinear in nature.

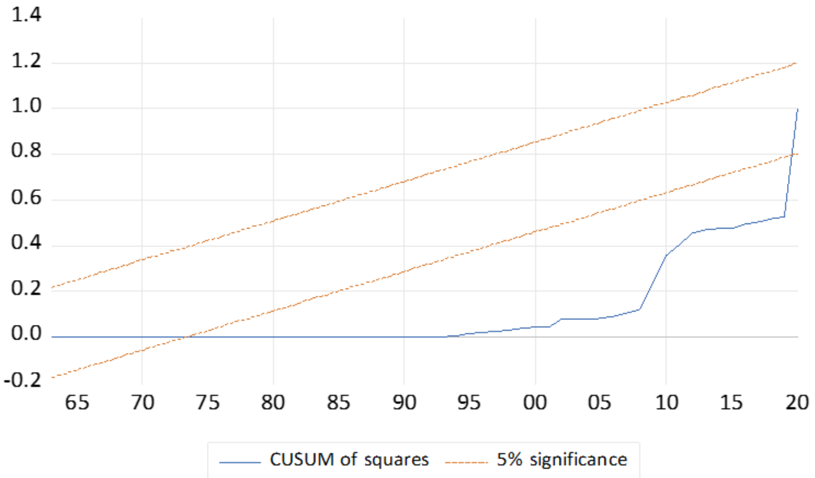


Figure 2. CUSUM of Squares test of the study.

4.1.5. Bierens Nonlinear Unit Root Results

Table 3 presents the Bierens nonlinear unit root test. The test consists of test statistics and simulated p-values for DBTM. The Akaike information criteria (AIC)are applied for selecting the optimal lag length, and the gaussian process is used to obtain test statistics results. The result reveals that at 5%, the null hypothesis ofanonlinear unit root cannot be rejected. The statistical values of the T(v), Am, and Fm-tests presented are all greater than their matching critical values, and this is expressed through p-values that are more than 5% significance level. In conclusion, this implies that public debt (DBTM) is non-stationary at certain levels. This paper proceeds with the modelling of the Markov Switching model (MS-AR).

Table 3. Bierens nonlinear unit root test for DBTM.

Variable	Test	Test statistics	Simulated p-values
DBTM	T (v)	3.952	0.500
	A(Am)	10.789	0.500
	A(Fm)	5.992	1.000

4.2. Markov Switching Model (MS-AR) Estimation Results

Before obtaining the final form of the model used in this study, various forms of lagged values of autoregression on public debt were considered. Table 4 compares the appropriateness of the various estimated two-state Markov switching models.

4.2.1. Model Selection

Table 4 demonstrates that in using some evaluation measures such as the Akaike information criteria (AIC) and log-likelihood, between the six estimated MS-AR(1) and MS-AR(6). The selected model is MS-AR(3) with the lowest Akaike information criteria (AIC) of 20.745 and a log-likelihood of -591.633.

Table 4. Best model selection.

MS-AR	Number of regimes	Log-likelihood	AIC
1	2	-619.311	20.843
2	2	-604.229	20.753
3	2	-591.633	20.745**
4	2	-581.082	20.809
5	2	-570.875	20.888
6	2	-563.257	21.063

Note: **/ 5% significance.

After adopting the model MS-AR(3),it was subjected to residual diagnostic tests. Then the tests of normality with Jarque-Bera and correlograms of the squared residuals on autoregressive conditional heteroskedasticity (ARCH) were applied. The Jarque-Bera test results indicate (see Appendix A) that the residuals were found not to be normally distributed. If the residuals are not normally distributed, this suggests that the histogram is not bell-shaped and the Jarque-Bera statistics are significant. The ARCH test (See Appendix B) reported no issues of homogeneity in the variance of the error term. This implies that if there is no evidence of ARCH in the residuals, then the autocorrelations and partial autocorrelations values must be zero at all lags, and the Q-statistics must be insignificant.

4.2.2. Model Estimation

Table 5 shows that all parameters estimated for the MS-AR (3) model are found to be significant at the conventional level. Furthermore, Table 6 shows the transition probabilities. The projected transition probabilities signify that there is a higher likelihood that the system stays in the same state, thus suggesting a few shifts in the regime. Empirically, the results show a 98% likelihood of remaining in high public debt and a lower likelihood of 1.76% switching to a lower public debt regime. Similarly, when the system is in a lower debt regime, there is a 98.2% likelihood of remaining in a lower public debt regime, and again, a lower possibility of 1.74% shifting to a higher public debt regime. This finding implies that only in an extreme event can public debt switch from a high debt regime to a lower debt regime, or oppositely behave.

Table 5. Estimation results of MS (2) – AR (3) model for the period 1960 – 2020.

Parameters	Regime 1	Regime 2
MS-AR(1)	1.42. (0.000)**	2.541 (0.000)**
MS-AR(2)	-0.693 (0.090)*	-2.318 (0.090)*
MS-AR(3)	0.475 (0.094)*	0.839 (0.094)*
LOG(SIGMA)	6.831 (0.000)**	10.514 (0.000)**
Transition matrix parameters		
P11-C	4.017	
	(0.003)**	
P21-C	-4.0302	
	(0.003)**	

Note: */ 10% significance and **/ 5% significance.

Table 6. Constant Markov transition probabilities and expected durations.

Constant Markov transition probabilities	Regime 1	Regime 2
Regime 1	0.982	0.018
Regime 2	0.017	0.983
Constant expected durations	56.547	57.275

The expected (average) duration of a period of higher public debt is 56 years, while the average duration of a lower public debt regime is 57 years. The implications of these expected durations explain why public debt in South Africa stays in either of the regimes for the same expected duration. These results differ from those of Olaoye and Olomola (2022), whose transition between both regimes is highly persistent. This is understandable by looking at the statistics of South Africa, where public debt has constantly been increasing over many years without fluctuations.

4.3. Graphical Representation of the Filtered Regime Probabilities

Figure 3 shows the graphical representation of the filtered regime probabilities. It is evident from the graph that the higher public debt in regime 1 is much more the same in regime 2, which is the lower public debt. This confirms the implication that only in an extreme event can public debt switch from a high-debt regime to a lower-debt regime. One of the objectives of this investigation is the forecasting ability of MS-AR(3). Estimating the future value of public debt is essential for the sake of fiscal policy decision-making and policy formulation. The Markov-switching model which is the MS(2)-AR(3), was estimated based on the yearly data from 1960 to 2020. The plot of the MS(2)-AR(3) forecasts is depicted in Figure 4. It is evident from the figure that the suggested model fits well with that data, as the forecasted values (DBTMF) mimic the actual data well.

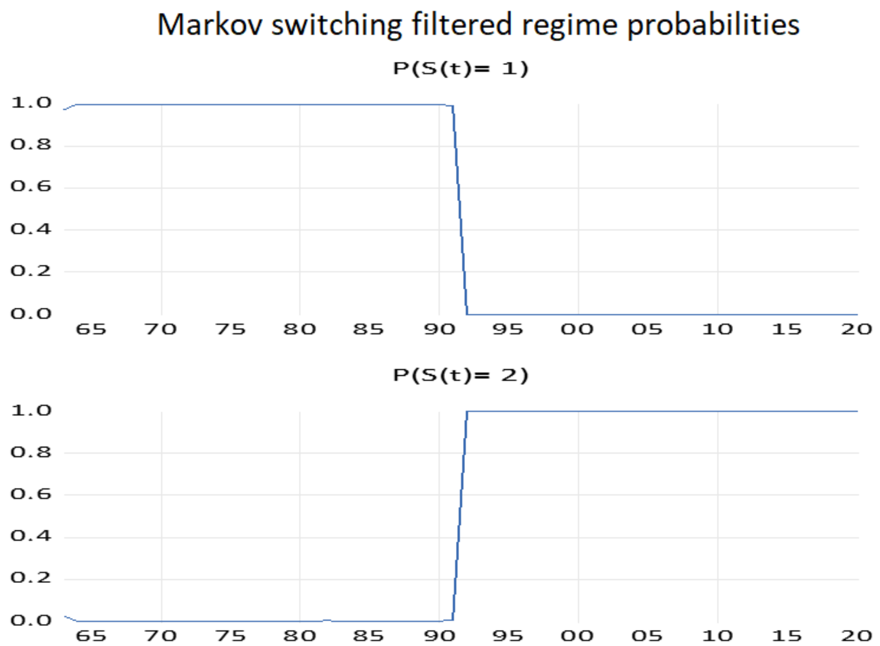


Figure 3. Graphical representation of the filtered regime probabilities.

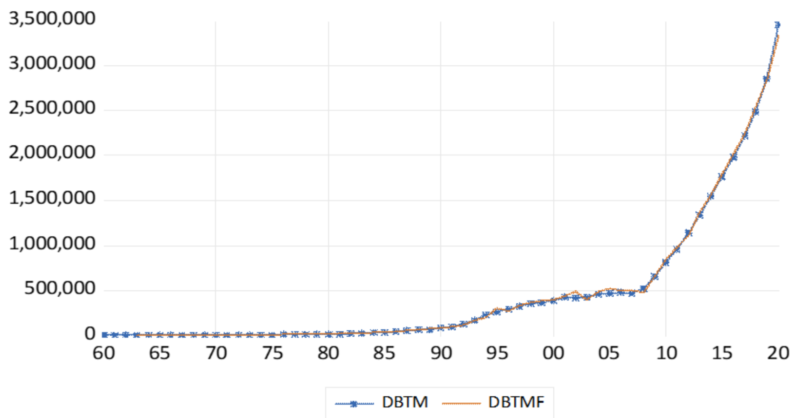


Figure 4. Forecasted and actual values of public debt.

4.4. Forecasting Evaluation

To examine the routine of the estimated model in predicting the future pattern of public debt, twoerror metrics were considered, which are Mean Absolute Percentage Error (MAPE) and Theil's U criteria. These two criteria, as shown in Table 7, incontrovertibly identify the fitted model MS(2)-AR(3) as a good model capable of forecasting the future values of South African public debt.

Table 7. Forecasting evaluation.

Test evaluation	Coefficients
MAPE	5.221
Theil's U	0.014

5. Conclusion of the Study

In this study, the South African public debt was estimated using two regime-switching autoregressive processes. One of the most critical matters for fiscal policy decision-making is that when they want to make decisions on stabilization policies, they must forecast the utmost possible period of the next regime shift of fiscal instruments. The MS (2)-AR (3) precisely projected the historical changing moments for public debt in South Africa. Empirically, the results show a 98% likelihood of remaining in high public debt and a lower likelihood of 1.76% shifting to a lower public debt regime. Similarly, when the function is in a lower debt state, there is a 98.2% likelihood of remaining in a lower public debt regime and a lower p-value of 1.74% shifting to a higher public debt regime. The expected (average) duration of a period of higher public debt is 56 years, while the average duration of a lower public debt regime is 57 years. This implies that the higher public debt in regime, 1 is much more the same in regime 2, which is the lower public debt.

The result of this study has important policy implications: only in an extreme event can public debt switch from a high debt regime to a lower debt regime and vice versa. Also, in South Africa, policymakers should keep in mind that either of the regimes will stay the same for about 56 years, only in the extreme event that public debt can switch from a high regime to a lower debt regime and vice versa.

The limitation of this paper is that the forecasted stock of public debt in South Africa was based on a univariate analysis of MS-AR. This can be viewed as a limitation if much of the South African public debt is influenced by other economic variables. To overcome this possible limitation, future studies should attempt to use more advanced forecasting methods, such as artificial neural networks (ANN), to strengthen the forecast of public debt. It is mentioned in the literature that sometimes MS-AR models might not be efficient for distinguishing the performance of dynamic time series that have moving average terms, which might lead to low forecasting ability.

References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control, 19*(6), 716-723. https://doi.org/10.1109/tac.1974.1100705

Baharumshah, A. Z., Jibrilla, A. A., Sirag, A., Ali, H. S., & Muhammad, I. M. (2016). Public revenue-expenditure nexus in South Africa: Are there asymmetries? *South African Journal of Economics, 84*(4), 520-537. https://doi.org/10.1111/saje.12130

Beqiraj, E., Fedeli, S., & Forte, F. (2018). Public debt sustainability: An empirical study on OECD countries. *Journal of Macroeconomics, 58*(C), 238-248. https://doi.org/10.1016/j.jmacro.2018.10.002

Bierens, H. J. (1997). Testing the unit root with drift hypothesis against nonlinear trend stationarity, with an application to the US price level and interest rate. *Journal of Econometrics, 81*(1), 29-64. https://doi.org/10.1016/s0304-4076(97)00033-x

Bohn, H. (1998). The behavior of US public debt and deficits.*The Quarterly Journal of Economics, 113*(3), 949-963. https://doi.org/10.1162/003355398555793

Broock, W. A., Scheinkman, J. A., Dechert, W. D., & LeBaron, B. (1996). A test for independence based on the correlation dimension. *Econometric Reviews, 15*(3), 197-235. https://doi.org/10.1080/07474939608800353

Burger, P., & Marinkov, M. (2012). Fiscal rules and regime-dependent fiscal reaction functions: The South African case. *OECD Journal on Budgeting, 12*(1), 1-29. https://doi.org/10.1787/budget-12-5k9czxjth7tg

Burger, P., Stuart, I., Jooste, C., & Cuevas, A. (2012). Fiscal sustainability and the fiscal reaction function for South Africa: Assessment of the past and future policy applications. *South African Journal of Economics, 80*(2), 209-227. https://doi.org/10.1111/j.1813-6982.2012.01321.x

Curtaşu, A. R. (2011). How to assess public debt sustainability: Empirical evidence for the advanced European countries. *Romanian Journal of Fiscal Policy, 2*(2), 20-43.

Debrun, X., Ostry, J. D., Willems, T., & Wyplosz, C. (2019). Debt sustainability. *Sovereign Debt: A Guide for Economists and Practitioners, 151*, 151-191. https://doi.org/10.1093/oso/9780198850823.003.0005

Djeutane, G. K., & Munditi, N. (2014). An overview of domestic debt in SADC: A synthesis of trends, structure and development impacts. African Forum and Network on Debt and Development (AFRODAD).

Domar, E. D. (1944). The" burden of the debt" and the national income. *The American Economic Review, 34*(4), 798-827.

Ganyaupfu, E. M. (2014). Fiscal sustainability in South Africa: Recent evidence. *Journal of Social Economics, 1*(4), 159-167.

Ganyaupfu, E. M., & Robinson, Z. C. (2019). Fiscal sustainability and interdependence of primary balance and public debt in South Africa. *Acta Universitatis Danubius Œconomica, 15*(3), 318-334.

Hakkio, C. S., & Rush, M. (1991). Is the budget deficit "too large?". *Economic Inquiry*, *29*(3), 429-445. https://doi.org/10.1111/j.1465-7295.1991.tb00837.x

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, 357-384. https://doi.org/10.2307/1912559

Kaur, B., Mukherjee, A., & Ekka, A. P. (2018). Debt sustainability of states in India: An assessment. *Indian Economic Review*, *53*(1), 93-129. https://doi.org/10.1007/s41775-018-0018-y

Lankester Campos, V., Loaiza Marín, K., & Monge Badilla, C. (2020). *Primary balance: Sustainability analysis under uncertainty No. IDB-WP-1151*. IDB Working Paper Series.

Mahmood, T., Arby, M. F., & Sherazi, H. (2014). Debt sustainability: A comparative analysis of SAARC countries. *Pakistan Economic and Social Review*, *52*(1), 15-34.

Naraidoo, R., & Raputsoane, L. (2015). Debt sustainability and financial crises in South Africa. *Emerging Markets Finance and Trade*, *51*(1), 224-233. https://doi.org/10.1080/1540496x.2015.1011534

Olaoye, O. O., & Olomola, P. (2022). Empirical analysis of asymmetry phenomenon in the public debt structure of Sub-Saharan Africa's five biggest economies: A markov-switching model. *The Journal of Economic Asymmetries*, *25*(C), e00242. https://doi.org/10.1016/j.jeca.2022.e00242

Redda, E. H. (2020). Sustainability of public debt and budget deficit in South Africa. *Journal of Management Information and Decision Sciences*, *23*(3), 164-174.

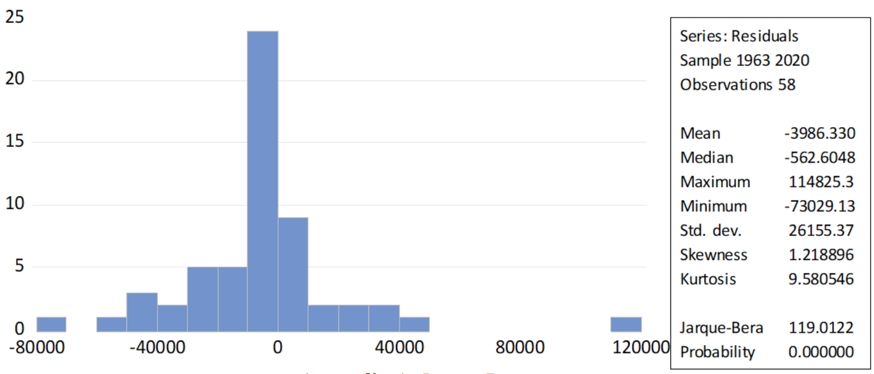
Saungweme, T., & Odhiambo, N. (2020). Causality between public debt, public debt service and economic growth in an emerging economy. *Studia Universitatis Babeş-Bolyai Oeconomica*, *65*(1), 1-19. https://doi.org/10.2478/subboec-2020-0001

South African Reserve Bank. (2022). *Publications*. Retrieved from https://www.resbank.co.za/en/home/publications

Stoian, A., Campeanu, E., & Roman, M. (2007). Fiscal sustainability based on reaction function: Case study Romania international confeence treaty on European Union, official macroeconomic analysis and international finance. In (pp. 24-26). Grecia: University of Crete, Rethymno.

Woldu, G. T. (2022). Do fiscal regimes matter for fiscal sustainability in South Africa? A markov-switching approach. *Applied Economics Quarterly*, *68*(1), 53-78. https://doi.org/10.3790/aeq.68.1.53

Appendix



Appendix A. Jarque-Bera test.

Q-statistic probabilities adjusted for 3 dynamic regressors

Auto correlation	Partial correlation		AC	PAC	Q-stat	Prob*
		1	0.241	0.241	3.5508	0.060
		2	0.144	0.091	4.8358	0.089
		3	-0.004	-0.061	4.8366	0.184
		4	-0.022	-0.024	4.8692	0.301
		5	0.036	0.060	4.9559	0.421
		6	-0.060	-0.080	5.1931	0.519
		7	-0.066	-0.054	5.4887	0.601
		8	0.049	0.104	5.6549	0.686
		9	0.021	0.002	5.6857	0.771
		10	-0.243	-0.310	9.9718	0.443
		11	-0.004	0.150	9.9729	0.533
		12	-0.090	-0.035	10.592	0.564
		13	-0.133	-0.217	11.953	0.531
		14	-0.093	-0.002	12.631	0.556
		15	-0.137	0.005	14.143	0.515
		16	0.079	0.052	14.659	0.550
		17	0.087	0.022	15.302	0.574
		18	-0.047	-0.045	15.496	0.628
		19	-0.042	-0.048	15.650	0.680
		20	0.083	0.060	16.281	0.699
		21	-0.038	-0.037	16.418	0.746
		22	-0.024	-0.081	16.476	0.792
		23	-0.103	-0.095	17.536	0.782
		24	-0.100	-0.061	18.567	0.775

*Probabilities may not be valid for this equation specification.

Appendix B. Correlogram Q- statistics.