



Mutual operator patterns in metaheuristics: A review of classical numerical optimization problems

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Abstract

Operator landscapes form an essential tool for evaluating the performance, stability, and convergence ability of metaheuristic algorithms in the face of modality, separability, dimensionality, ruggedness, and landscape deception. For this reason, state-of-the-art hybrid metaheuristics are more often than not built based on the use of several search operators. Despite a growing number of scientific works devoted to various aspects of hybrid optimization, there remains a lack of information about the mutual patterns of operator cooperation that have helped achieve optimization results. To address this gap, this paper reviews common patterns of operator cooperation in metaheuristics used for the classical problem of numerical optimization. First, recurring patterns of operator cooperation in the context of numerical optimization problems, which involve both unimodal, multimodal, rotated, shifted, hybrid, composition, and high-dimensional benchmarks, are discussed. Second, common operator types used in modern optimization algorithms are introduced. Lastly, common patterns of operator cooperation involving combinations of operators used for exploration and exploitation of the optimization landscape, diversification and perturbation of solutions, chaos-based search, Lévy-flight diversification, memetic search, operators' adaptation to the current optimization landscape, and simultaneous multi-population optimization are discussed. In particular, operator-oriented taxonomy and analysis of the ways in which operator cooperation depends on the properties of the landscape are proposed. Key areas of further research include operator compatibility, scalability, reproducibility, explainability, and operator autonomy. New trends in the field consist of adaptable operator ecosystems, reinforcement learning-assisted optimization, modular structure of hybrid algorithms, and self-evolution. In conclusion, the significance of operator cooperation in achieving good optimization results is highlighted.

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1. Introduction

Optimization plays a key role in modern day scientific areas such as computer science [1], engineering [2], artificial intelligence [2], logistics [3], manufacturing [4], and decision making [5]. The goal of optimization is to find optimal solutions for problems involving nonlinear and computationally complex scenarios. Typically, optimization problems involve qualities like multi-modality, high dimensions,

inter-dependency of variables, discontinuity, and deceptiveness. Such properties in optimization problems often make traditional deterministic approaches [1] inadequate for obtaining solutions. Meta-heuristics on the other hand have been successful owing to their non-differentiable approach and proven results over various kinds of optimization problems [1, 6, 7].

Numerical test benchmarks have an important place in determining the effectiveness of metaheuristics, as they provide artificial optimization environments whose features can be altered in terms of their modality, separability, dimensions, ruggedness, and rotational structure [7, 8]. The frequently used test function sets consist of unimodal test functions like Sphere and Zakharov functions; multimodal test functions like Rastrigin, Ackley, Schwefel, and Griewank; and rotated, shifted, hybrid, and composite test functions which are usually used in the CEC benchmark problems competition [9]. These artificial optimization test function landscapes are designed to test the following:

- Exploration capability.
- Exploitation efficiency.
- Convergence stability.
- Robustness.
- Scalability.
- Stagnation resistance.
- Adaptation to complex fitness landscapes.

In the recent past, many different meta-heuristic methods have been designed based on the principles of biological evolution, swarms, natural phenomena, sociological behaviour, and biology. Some popular methods which have been used for solving nonlinear multimodal optimization problems are Genetic Algorithms (GA) [10], Particle Swarm Optimization (PSO) [11], Differential Evolution (DE) [12], Grey Wolf Optimizer (GWO) [13], Whale Optimization Algorithm (WOA) [14], and Artificial Bee Colony (ABC) [15]. These algorithms have demonstrated strong capability for solving nonlinear and multimodal optimization problems across numerical optimization benchmarks and real-world applications [1].

Although a number of significant progresses have been made through metaheuristic approaches, recent studies reveal that there is a trend toward devising many algorithms based on existing metaphors because they utilize already existing heuristic search procedures [16]. In addition, recent literature shows that innovation involves a specific metaphor but uses the same search procedure [16].

Another reason that makes the hybridization of metaheuristics quite important comes from the No Free Lunch (NFL) theorem, according to which no optimization technique is able to perform better than other techniques for any type of problem encountered [17]. The efficiency of the optimization technique will therefore depend upon the ability of the technique to adapt to the problem at hand [18]. Optimization techniques include.

- Global exploration operators.
- Local exploitation procedures.
- Lévy-flight perturbation.
- Chaotic diversification.
- Adaptive parameter scheduling.
- Restart strategies.
- Opposition-based learning.
- Reinforcement-learning-guided operator selection.
- Hyper-heuristic scheduling frameworks [19].

Further evidence of this shift is offered by recent developments in adaptive optimization and coordination of intelligent search processes. Specifically, hyper-heuristics and machine-learning supported optimization methods are now being employed more and more to dynamically choose and coordinate low-level search heuristics according to convergence properties, diversity characteristics, and information obtained from the process of optimization [19]. Similarly, current research on modular hybrid metaheuristic approaches also shows that there is an increasing focus on creating reusable operator environments and adaptive hybridization structures that automatically design optimization strategies [20].

While hybridization has become a key trend in recent optimization investigations, existing review literature generally classifies optimization strategies based on algorithms, metaphors, or problem domains [1, 6]. There has not been much focus on analysing the interactions among operators used by various optimization methodologies, which appear frequently in many optimization models and benchmark problems. The existing surveys mostly consider performance comparisons, convergence rates, or rankings of benchmark problems without:

- Operator compatibility.
- Landscape-aware hybridization.
- Adaptive operator coordination.

- Recurring operator interactions across numerical landscapes.

This problem is particularly prominent in classical numerical optimization, whereby benchmarks often have their own structural attributes. These include, but are not limited to:

- Unimodal landscapes for evaluating exploitation abilities.
- Multimodal landscapes for assessing diversification abilities.
- Rotated and non-separable landscapes for assessing robustness.
- Hybrid/composite problems that require multiple operators to cooperate.

As such, some numerical landscapes could accommodate several types of interaction between operators.

Addressing a significant gap in the literature, the paper provides an all-encompassing operator-oriented study of the commonalities among the patterns of operators that have proven effective within metaheuristic approaches to classical numerical optimization problems. Instead of focusing on algorithm families and their metaphoric inspiration, the focus is on patterns of operators that are common to effective optimization frameworks. The study covers:

- Exploration operators.
- Exploitation processes.
- Diversity maintenance techniques.
- Perturbation operators.
- Adaptation processes.
- Learning-enabled search processes.

Main objective of the current review includes detection of common operator-based patterns in optimization frameworks and the analysis of such operator patterns in relation to particular optimization landscape features, which include modality, dimensionality, separability, ruggedness and deception of the landscape. In addition, recent advances in the field include:

- Hyper-heuristics.
- Reinforcement learning-based operator scheduling.
- Machine learning-based optimization.
- Adaptive operator ecosystems.
- Self-adaptive hybrid optimization algorithms.

Principal contributions of this literature review can be summarized as follows:

1. Development of a new operator-focused approach to the investigation of hybrid metaheuristic algorithms in numerical optimization problems.
2. Identification of common mutual operator patterns in unimodal, multimodal, rotated, shifted and composite optimization functions.
3. Development of a unified taxonomy of exploration, exploitation, diversity-preserving, adaptive and perturbation operators in state-of-the-art optimization algorithms.
4. Examination of relations between properties of benchmark optimization landscapes and operator-based hybrid algorithms.
5. Identification of emergent research trends within adaptive operator ecosystems, intelligent operator interactions, hyper-heuristics and autonomous optimization.

The rest of the manuscript proceeds as follows. In Section 2, an overview of the main categories of classical optimization problems along with their benchmarking functions is presented. In Section 3, the theoretical background of operator-level hybridization, as well as the most common operators used in numerical optimization, is elaborated upon. In Section 4, an analysis of the most frequently occurring operator patterns described in optimization literature is provided. In Section 5, operator categorization approaches are compared, whereas the correspondence between the optimization problem and the operators used to solve it is considered. In Section 6, the consequences of the observed operator structure on optimization studies are discussed. In Section 7, several open questions regarding operator-level hybridization are listed, and in Section 8, future directions in adaptive optimization ecosystems are analysed.

2. Classical Numerical Optimization Landscapes

The use of numerical benchmark optimization problems with their well-known structural properties including modality, separability, dimensionality, ruggedness, rotational invariance, and deceptiveness [1, 6] is considered to be standard practice in the domain of metaheuristic algorithms owing to the availability of an easily controllable problem space. The benchmark optimization problems offer the advantage that they are ideal testing grounds for carrying out experiments to understand aspects of the search process, especially when compared to the real-life optimization problems. It follows, then, that benchmark test functions have come to play an important role in recent optimization research to study issues of diversification, local search, adaptive control, and convergence [1, 6].

The classical benchmark functions include the following major groups according to the optimization problem difficulty level and corresponding landscape characteristics.

- Unimodal landscapes.
- Multimodal landscapes.
- Shifted and rotated functions.
- Hybrid and composition functions.
- Multi-dimensional optimization problems.

Different levels of search problem difficulty require different approaches towards developing new operator-based hybrid techniques.

2.1. Unimodal Optimization Landscapes

Unimodal benchmark functions, which have just one global optimum, are used mainly to check how well algorithms can find and improve upon good solutions [1, 6]. Because these functions mostly feature smooth surfaces with hardly any local minima, the focus is on seeing how quickly optimizers can converge and how stably they perform numerically, plus how they enhance solutions locally.

Unimodal benchmark functions include the following: the Sphere, the Zakharov, the Sum Squares, the Schwefel 2.22, and the Rosenbrock benchmark functions. Of these benchmark functions, the Sphere and the Sum Squares functions are used to evaluate convergence behaviour while the Rosenbrock benchmark function has curved valleys and presents challenges not just with respect to convergence but with local search also [19, 21].

2.2. Multimodal Optimization Landscapes

Multimodal functions have several local optima. These functions are used to evaluate the performance of optimization algorithms with respect to exploration and prevention from local optima [1, 6]. These functions present a difficult task for optimization algorithms compared to unimodal functions, due to their tendency to converge towards suboptimal solutions.

Benchmarking functions that have been used in the past to evaluate multimodal functions include Rastrigin, Ackley, Griewank, Schwefel, Alpine, and Michalewicz functions. This is because these functions have proven to be difficult to solve on account of being rough, periodic, deceptive and having complex basins of attraction. Hence, any optimization strategy used to solve such functions must necessarily include a combination of various strategies. These are diversification for global exploration, Levy flights, chaos-based search strategy, restarting for avoidance of local minima, opposite based learning for better initialization, and balance between exploration and exploitation [2, 21, 22].

However, recent research reveals that there has been a rise in the use of optimization algorithms which maintain population diversity and adapt the operators dynamically as the problem is being solved. This is explained in Zhang, et al. [22] and Juárez, et al. [19].

2.3. Rotated and Shifted Landscapes

This represents a difficulty since the transformations have been subjected to rotations as well as translations, changing the geometry of the function itself [1, 6]. This creates a difficulty due to the fact that separating variables becomes impossible, along with making the identification of the global optimal point ambiguous [1, 19].

Some of the functions which have made an impact on this area are the Rotated Rastrigin Function, the Shifted Rosenbrock Function, the Rotated Ackley Function, the Shifted Schwefel Function, and the Rotated Griewank Function.

For overcoming these problems, quality optimization algorithms may include such features as parameters' tuning, covariance modification, handling of many populations, and special mutations for differential evolution [1, 19]. Also crucial is the ability of the algorithm to react to the data obtained from the environment. Since random traversal of the space undermines the fundamental search technique based on coordinate axes, optimization needs a well-coordinated interaction between operators [1, 19].

2.4. Hybrid and Composition Landscapes

The application of hybrid and composite benchmark functions in optimization poses serious obstacles. With their ability to aggregate a wide variety of complex properties in one function, they make the optimization process very hard [1, 6]. The reasons behind the complexity of the functions include multimodality, interaction of variables, non-separability, rotations, diverse scaling, rugged fitness landscape and deceptive paths that lead the algorithms astray [2, 19].

Such functions are used for testing the abilities of optimization methods in very complex situations. Examples of them include CEC hybrid functions CF1–CF10 and large-scale composite benchmark functions.

Unlike in simple tests, such functions impose requirements on an algorithm to handle a number of components simultaneously. In particular, a good method should explore the space of solutions, ensure accuracy, adjust to the conditions, when necessary, adopt good restarting techniques and have appropriate

control over search direction [2, 19]. It is because of these reasons that studies of these complex functions result in the development of adaptive operators and search strategies.

2.5. High-Dimensional Optimization Landscapes

The optimization methods are verified on such large and high dimensional landscapes when the problem becomes harder [1, 6]. Moving into higher dimensional search space leads to problems like population collapse, premature convergence, stagnation in search and increased computational cost [1, 6].

For verification, benchmark functions are employed to evaluate the capability of the methods to overcome the above-mentioned problems through conducting experiments in dimensions of 50, 100, 500 and 1000. Methods that address high dimensions include multi-population models, adaptive dimension interaction, cooperative co-evolution, distributed search and large-scale diversity preservation operators [1, 6].

It is evident from the discussion above that the methods depend on adaptive operator scheduling and machine learning based search coordination [2, 19].

2.6. Comparative Characteristics of Numerical Optimization Landscapes

Despite the different levels of complexity and structures, classical numerical benchmark problems remain useful means for investigating hybrids and search processes on the operator level. These characteristics are described in Table 1 of this essay. The mentioned instruments have found wide application and contributed to the development of hybrid metaheuristics, adaptive optimization techniques, and intelligent coordination of search processes [1, 6, 19]. Above all, the common elements applied while solving such benchmarks allow researchers to start from here when looking for analogies in modern numerical optimization studies.

Table 1. Comparative characteristics of classical numerical optimization landscapes.

Landscape Type	Main Difficulty	Typical Operator Requirement
Unimodal	Convergence precision	Local exploitation
Multimodal	Premature convergence	Diversification
Rotated/Shifted	Non-separability	Adaptive coordination
Hybrid/Composition	Complex landscape interaction	Multi-operator cooperation
High-dimensional	Scalability	Population coordination

3. Operator-Level Hybridization

The concept of operator-level hybridization refers to the fusion of diverse search strategies derived from different optimization methods with the purpose to increase their efficiency, robustness and flexibility [1, 16]. The fusion itself is not about combining algorithms but rather about combining the basic search operations such as perturbation, improvement of the local solution, restarts and diversity preservation [2, 19].

It is important in classical numerical optimization due to the existence of benchmark problems with very different structure and difficulty, which requires proper approaches for their solution [1, 6]. The simplest problems with only one peak do not require complicated strategies but problems with multiple peaks or complicated structure require more advanced approaches which will ensure broad searches and avoiding of being trapped in local minima [2, 19].

When dealing with extremely difficult optimization problems, it becomes necessary to fuse the different processes in order to provide proper balance between exploration, exploitation, adaptability and diversity [1, 6].

The numerical optimization literature reveals five major types of operators.

- Exploratory operators.
- Exploitative operators.
- Diversity-preserving operators.
- Perturbation operators.
- Adaptive operators.

These bits come together to create the core framework for modern hybrid metaheuristics and adaptive systems used in optimization.

3.1. Exploration Operators

The exploration operators provide an in-depth analysis of the problem domain and help to explore wide areas in search of good solutions [1, 21]. The main goal of the exploration operators is to increase the scope of analysis and avoid early convergence of algorithms on sub-optimal solutions. These operators are especially important in cases when the problem involves many local optima, deception, and high-dimensional spaces.

Examples of such operators include the differential evolution mutation operator, swarm intelligence-inspired movement operators, random walk operators, opposition-based learning operators, population

migration operators, and spiral search operators. Modern methods of optimization usually combine exploration operators with adaptive operators that are used to modify these operators in order to increase diversity of solutions [2, 22].

3.2. Exploitation Operators

The use of algorithms helps to make the search procedure faster and more efficient [1, 19]. Algorithms are used to make the solution better after the areas where possible solutions exist have been found. These algorithms include hill climbing algorithm, simplex algorithm, variable neighbourhood search (VNS), simulated annealing algorithm, local search algorithm, and gradient search.

It is clear that these algorithms are suitable not only for finding solutions to relatively easy tasks but also for tuning parameters at later steps of the process of optimization [8]. Currently, there are many methods that involve exploiters within population-based systems.

3.3. Diversity-Preservation Operators

The main idea of diversity-preservation operators is to avoid population degradation and degeneration during optimization [1, 22]. Preservation of diversity is extremely important for complex multimodal and deceptive search spaces because inadequate diversification can lead to early convergence to non-optimal local minima.

Popular diversity-preservation approaches are:

- Chaos-based perturbations.
- Perturbations via mutation.
- Crowding.
- Fitness sharing.
- Restarts.
- Population reinitialization.
- Cooperation between populations.

It has been shown that usage of diversity-preservation operators provides robustness and convergence stability on rough optimization landscapes [2, 21, 22]. Therefore, modern optimization algorithms are able to automatically control the level of diversification based on convergence and population dynamics.

3.4. Perturbation Operators

A perturbation operator adds disturbances to the optimization process to enhance the capability of escaping from the local optimum and increasing the flexibility of searching [1, 2]. Different from general exploration operators, the perturbation approach is specially used for creating some deviated behaviours to existing search procedures.

Some commonly used perturbation operators are as follows:

- Lévy flight perturbation.
- Gaussian mutation.
- Chaotic perturbation.
- Random restarts.
- Brownian motions.
- Adaptive random walk.

In all these approaches, Lévy flight perturbation has gained much popularity because of the power-law distribution of its steps that can perform both local searching and global exploring [2, 21].

3.5. Adaptive Operators

Adaptive operators adaptively alter the optimization dynamics dependent on convergence, diversity, state feedback, and landscape dynamics [2, 19]. The primary objective of these operators is to guarantee the robustness of an optimization algorithm through dynamic adjustment of search effort and operators in different optimization phases.

Examples of adaptive operators are:

- Adaptive parameter tuning.
- Self-adaptive mutation.
- Adaptive crossover.
- Operator selection by means of reinforcement learning.
- Hyper-heuristic scheduling.
- Adaptive population management.

It has been observed that there is an emerging trend in modern optimization literature where scheduling schemes based on machine learning are being increasingly used to dynamically select operators according to

the feedback provided by the optimization and convergence dynamics [2, 19]. Adaptive coordination strategies have gained utmost importance in addressing benchmark landscapes with multimodal, rotational, non-separable, and high-dimensional nature.

3.6. Generalized Operator-Level Hybridization Architecture

In this modern optimization procedure, an idea of a multi-operator framework has been used, which is based on cooperation of exploration, exploitation, diversification, perturbation and adaptive cooperation during each stage of optimization [1, 16]. Differing from the traditional methods that are based on a certain search method, hybrid methods combine various operators for achieving stability in search.

The scheme of hybridization on the operator level is presented in Figure 1.

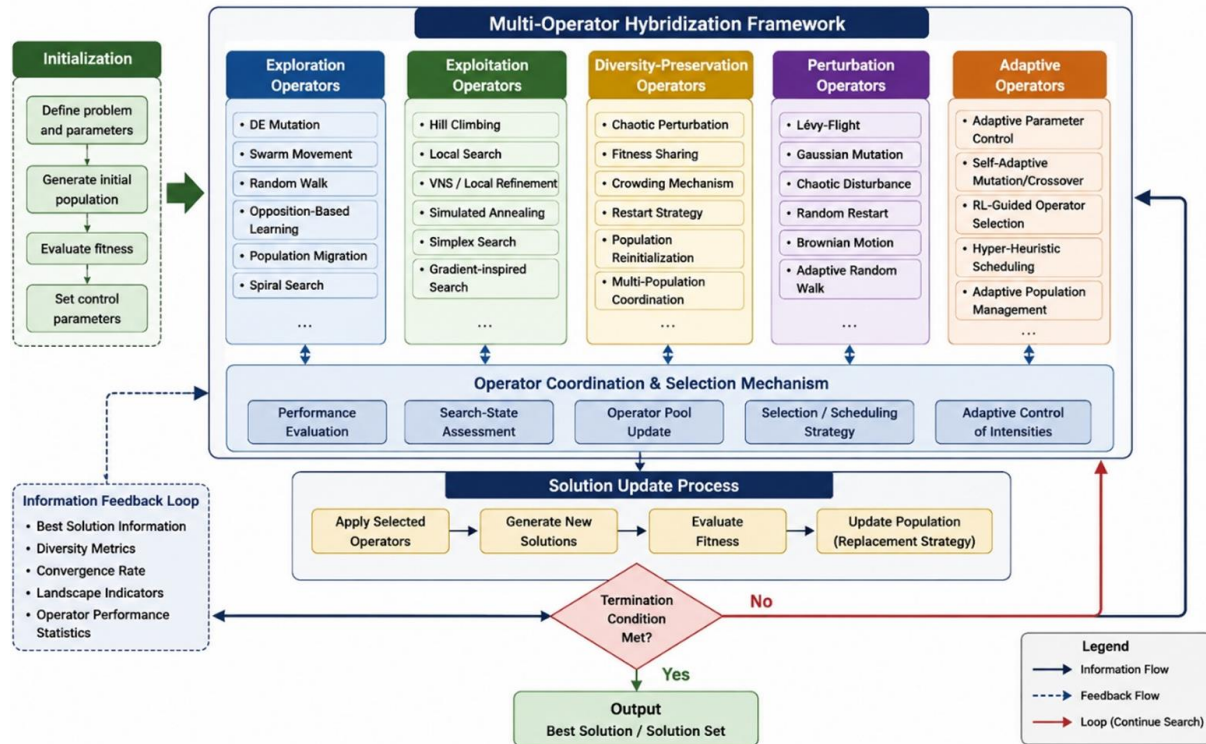


Figure 1. Generalized operator-level hybridization architecture.

4. Mutual Operator Patterns

4.1. Exploration–Exploitation Cooperation

Another common theme in the literature regarding optimization is the balancing of the operator's exploration and exploitation, which means that the diversification approach works in conjunction with the local search approach [1, 19, 21]. The exploration operators can be: Differential evolution mutation, swarm moves, opposition based learning, and random walk. On the other hand, local search approaches may be as follows: Hill climbing, simplex method, simulated annealing, and variable neighbourhood search.

This kind of hybrid structure appears quite often in the following problems:

- Multimodal problems.
- Rotated problems.
- Composite problems.
- High dimensional optimization problems.

When dealing with multimodal problems like Rastrigin, Ackley and Schwefel functions, a strong exploration capability is needed to prevent premature convergence of the algorithm during the initial phase of the search process, while the local search becomes more and more relevant as we approach the region of attraction [2].

4.2. Diversification–Perturbation Cooperation

A third type of operator paradigm involves combining diversification techniques with perturbation operators for improving escape capability from local optima [2, 22]. In such a configuration, population diversification operators work along with perturbation schemes that generate intentional deviation from existing paths of search process. Typical combinations used in practice involve.

- Chaotic diversification together with Lévy-flight perturbation.
- Mutation burst in conjunction with random restart.
- Opposition based learning with Gaussian mutation.
- Adaptive random walk with population reinitialization.

Such hybrid structures have been found highly effective in:

- Deceptive problem spaces.
- Rugged multimodal optimization problems.
- Optimized problem spaces.
- Multidimensional benchmark problem instances.

It has been demonstrated recently that diversification enhanced by perturbation techniques significantly improve stagnation resistance and global search capability of algorithms [2, 22].

4.3. Chaos-Assisted Search Coordination

Chaotic operators form a widely accepted improvement operator for modern hybrid metaheuristics [22]. Several well-known chaotic maps such as Logistic, Tent, Sine, Chebyshev, and Gauss maps are used for the purpose of improving population initialization, increasing diversity, allowing adaptive perturbation and controlling the state of the search.

The main applications of chaos-based optimization algorithms include.

- Multimodal optimization.
- Rotated problems.
- Hybrid composite functions.
- Large dimensional space.

Some recent comparison studies have shown that chaos-based searching techniques help.

- To increase population diversity.
- To explore the search space.
- To ensure stable convergence.
- To escape from local optima [22].

Due to the deterministic nature of chaotic perturbations which provide a random search process, chaos-based optimization algorithms tend to show more robustness in a complex environment than stochastic algorithms.

4.4. Lévy-Flight Hybridization Patterns

The Lévy-flight perturbation forms a recurring operator paradigm in numerical optimization literature [2, 21]. The operators that follow Lévy distribution exhibit heavy-tailed step sizes, which allows both short-range exploration and infrequent long-range exploration to occur. This operator paradigm is commonly found in:

- Differential evolution schemes.
- Swarm-based optimization.
- Adaptive population methods.
- Memetic metaheuristics.
- Hybrid optimization compositions.

Lévy-flight diversification is highly beneficial for:

- Multimodal spaces.
- Deceptive basins.
- Rugged search spaces.
- Large-scale optimization tasks.

According to recent studies, Lévy-flight perturbation improves the robustness of exploration and helps avoid stagnation due to the ability of the optimization system to get out of deep basins without losing exploitation potential [2].

4.5. Memetic Local Refinement Patterns

Memetic optimization frameworks combine the concepts of population-based global search with local search methods to improve convergence accuracy and exploitation ability [1, 19]. In this context, exploration techniques first determine potentially good regions in the search space, and local search techniques then boost the process of optimization in the neighbourhood of good solutions.

Well, received local enhancement methods used are:

- Hill climbing.
- Simplex search.
- Variable Neighbourhood Search.

- Local perturbation.
- Simulated annealing.

The application of memetic hybridization proves to be especially efficient for:

- Unimodal functions.
- Functions with curved valleys.
- Rosenbrock-type functions.
- Multimodal functions demanding high local precision.

Memetic enhancement techniques have been found to be effective in improving convergence and reducing the problem of oscillations in the later stages of the optimization process [21].

4.6. Adaptive Operator Scheduling Patterns

Operator scheduling adaptation represents a key issue in today's literature on optimization [2, 19]. Instead of relying on inflexible predefined strategies for activating operators, modern optimization techniques usually change their searching strategies based on the following considerations.

- The speed of convergence.
- Diversity constraints.
- Search state information.
- Problem landscape features.
- Performance data for operators.

Typical adaptive scheduling techniques comprise.

- Adaptive parameter control.
- Self-adaptive mutation.
- Adaptive population control.
- Reinforcement learning-based operator selection.
- Hyper-heuristic coordination schemes.

Such adaptive approaches are of special significance for:

- Hybrid landscapes.
- Rotated functions.
- Compositional benchmarks.
- Large-scale optimization tasks.

In which optimization needs keep changing during the search procedure. Recent studies have also shown that adaptive scheduling provides considerable robustness and stability gains for search processes in extremely dynamic optimization settings [2, 19].

4.7. Multi-Population Coordination Patterns

Multi-population coordination forms a vital form of hybridization in numerical optimization [1, 2]. Unlike the classical approach where there is only one evolving population, multi-population techniques involve splitting the search space into several interacting populations. Subpopulation interactions may take place by way of:

- Exploration-based search.
- Exploitation-based search.
- Perturbation-based search.
- Adaptation-based coordination.

Subpopulation coordination in multi-population techniques is accomplished using sharing information strategies like migration, elite exchange, adaptive subpopulation, and cooperative co-evolution. Multi-population hybridization proves highly efficient in:

- High dimensional search space.
- Large scale optimization.
- Multimodal optimization landscapes.
- Complex benchmark problems.

The exploration of different parts of the search space at once by multiple subpopulations makes the process both efficient and more robust to stagnation.

4.8. Generalized Mutual Operator Structures

However, there are some aspects that show that some algorithms have similar features, regardless of different metaphors used for designing such algorithms [1, 7, 16]. These algorithms have different structures but still they demonstrate the following interactions among their elements.

- Global search.

- Local search.
- Diversity.
- Cooperative search.
- Stagnation avoidance.

These interactions define the structure of modern meta-heuristics used to solve numerical optimization problems. The structure of the operators used for solving numerical optimization problems is presented in Figure 2.

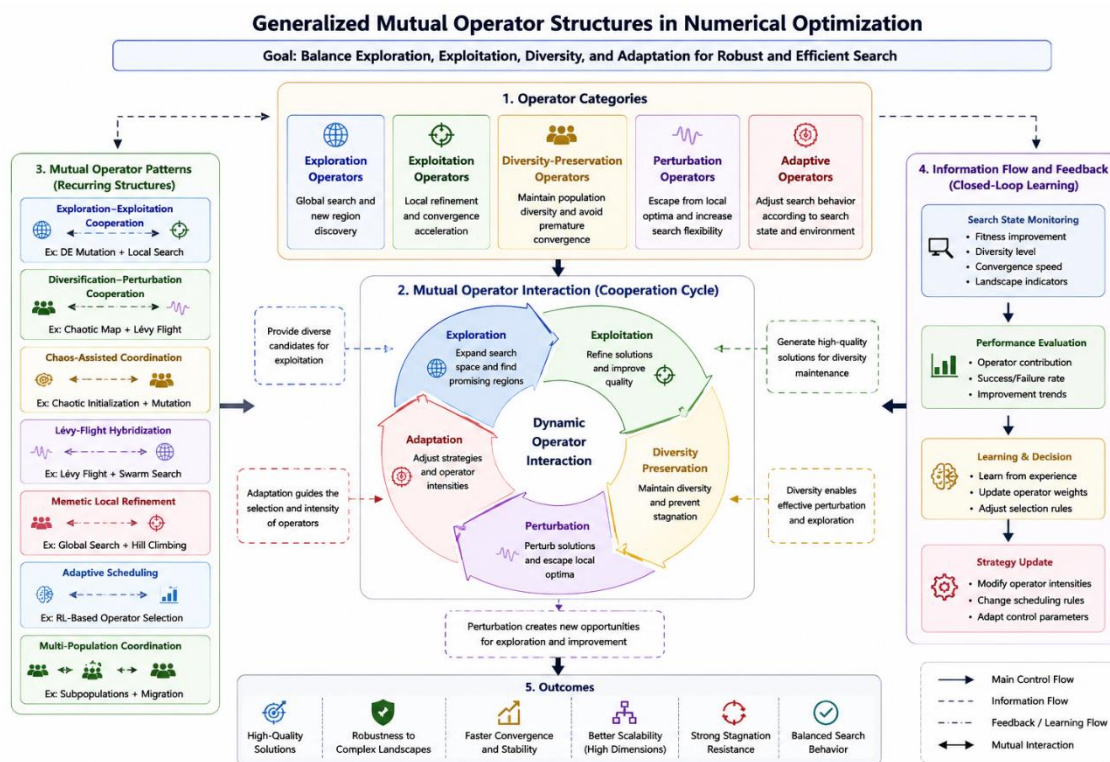


Figure 2. Generalized mutual operator structures.

5. Comparative Operator Taxonomy

In this section, we describe a brief and comparative classification of the main operator types that have been studied in numerical optimization as well as their usage for different classes of landscapes [1, 16]. While Section 4 has focused on structural issues of hybridization, this section is devoted to a functional and landscape-oriented classification of operators. Operator hybridization can be classified into five basic types of hybridizations:

1. Exploration operators.
2. Exploitation operators.
3. Diversity-preserving operators.
4. Perturbation operators.
5. Adaptive operators.

The tables below represent basic properties of these operators.

5.1. Exploration Operator Taxonomy

Exploration operators are essential in conducting global searches in large-scale landscapes. The purpose of exploration operators is to expand the space of the search landscape and reduce the probability of convergence during optimization processes, which can be multimodal or deceptive in nature. Exploration operators become even more important when dealing with multimodal and rugged search landscapes, as lack of diversification results in quick convergence towards local minima [2, 21].

Table 2. Exploration operator taxonomy.

Exploration Operator	Main Function	Suitable Landscapes
DE Mutation	Global perturbation	Multimodal
Swarm Movement	Population exploration	General optimization
Random Walk	Stochastic search expansion	Rugged landscapes
Opposition-Based Learning	Search-space expansion	Rotated functions

Population Migration	Region redistribution	High-dimensional
Spiral Search	Nonlinear exploration	Composition functions

5.2. Exploitation Operator Taxonomy

The exploitation operators focus their attention on optimizing the local area and ensuring convergence after the suitable search regions have been identified. The exploitation operators are highly efficient in cases where the environment is unimodal and locally reactive, and the significance of achieving accurate convergence increases at every stage [19, 21].

Table 3. Exploitation operator taxonomy.

Exploitation Operator	Main Function	Suitable Landscapes
Hill Climbing	Local refinement	Unimodal
Simplex Search	Precision convergence	Rosenbrock-type
Variable Neighbourhood Search	Adaptive local exploitation	Hybrid functions
Simulated Annealing	Controlled local search	Multimodal
Local Perturbation	Fine-grained adjustment	Composition functions
Gradient-Inspired Search	Convergence acceleration	Smooth landscapes

5.3. Diversity-Preservation Operator Taxonomy

Diversity-preservation operators maintain population variation and reduce stagnation during optimization.

Table 4. Diversity-preservation operator taxonomy.

Diversity Operator	Main Function	Suitable Landscapes
Chaotic Perturbation	Diversity enhancement	Multimodal
Fitness Sharing	Population spreading	Rugged landscapes
Crowding Mechanism	Maintain diversity	Deceptive functions
Restart Strategy	Escape stagnation	Local optimum traps
Population Reinitialization	Diversity recovery	High-dimensional
Multi-Population Coordination	Distributed exploration	Large-scale optimization

Recent optimization frameworks increasingly employ adaptive diversity management strategies to maintain search robustness in complex landscapes [2, 22].

5.4. Perturbation Operator Taxonomy

The perturbation operators create disturbances that increase the ability to escape from local minima. The Lévy flight perturbation is considered one of the most used mechanisms as a diversification operator in modern hybrid metaheuristic algorithms [2, 21].

Table 5. Perturbation operator taxonomy.

Perturbation Operator	Main Function	Suitable Landscapes
Lévy-Flight Perturbation	Long-distance jumps	Multimodal
Gaussian Mutation	Local stochastic perturbation	Smooth landscapes
Chaotic Disturbance	Irregular perturbation	Rotated functions
Brownian Motion	Continuous random variation	General optimization
Random Restart	Basin escape	Deceptive landscapes
Adaptive Random Walk	Dynamic perturbation	Hybrid functions

5.5. Adaptive Operator Taxonomy

These operators alter the process of search depending on the state of optimization.

Table 6. Adaptive operator taxonomy.

Adaptive Operator	Main Function	Suitable Landscapes
Adaptive Parameter Control	Dynamic search balancing	General optimization
Self-Adaptive Mutation	Automatic perturbation adjustment	Multimodal
Adaptive Crossover	Dynamic recombination control	Evolutionary search
RL-Guided Operator Selection	Intelligent scheduling	Composition functions
Hyper-Heuristic Scheduling	Operator coordination	Hybrid landscapes
Adaptive Population Management	Dynamic diversity control	High-dimensional

Adaptive operators are particularly important in rotated, shifted, hybrid, and composition functions, where optimization requirements vary continuously during the search process [2, 19].

5.6. Landscape-to-Operator Mapping

Different numerical landscapes frequently favour different operator categories according to modality, dimensionality, separability, ruggedness, and convergence sensitivity. Table 7 summarizes the dominant operator requirements for the major benchmark landscape categories discussed in this review.

Table 7. Landscape-to-operator mapping.

Landscape Type	Main Difficulty	Dominant Operator Requirements
Unimodal	Convergence precision	Exploitation
Multimodal	Premature convergence	Exploration + diversity
Rotated/Shifted	Non-separability	Adaptive coordination
Hybrid/Composition	Complex interaction	Multi-operator cooperation
High-dimensional	Scalability	Population coordination + adaptation

The comparative taxonomy indicates that successful numerical optimization frameworks typically combine multiple operator categories cooperatively according to landscape characteristics and optimization-state conditions [1, 2, 19]. More importantly, the repeated appearance of similar operator combinations across different benchmark categories further supports the existence of recurring mutual operator patterns in modern hybrid metaheuristics.

6. Discussion

Based on the analysis of the reviewed literature, it is clear that different numerical landscapes support different operator coordination approaches depending on such parameters as modality, separability, dimensionality, ruggedness, and convergence sensitivity [1, 2, 19]. In particular, unimodal problems are known to be advantageous for robust local refinement and convergence-driven exploitation approaches, while multimodal and deceptive landscapes require a more sophisticated approach that incorporates more diversification and stagnation prevention in order to maintain efficient global optimization capabilities. The rotated and non-separable landscapes increase the significance of the operator coordination capability even further.

Another significant observation relates to the growing importance of adaptive coordination in the face of optimization complexity. There exist many modern approaches that dynamically regulate operator effort, perturbation rate, diversity preservation, and search schedules depending on convergence dynamics and optimization state feedbacks. The most typical examples can be seen in reinforcement learning-based scheduling, hyper-heuristics coordination, self-adaptation of parameters, and optimization systems employing machine learning [2, 19].

Scalability is an important issue in current numerical optimization practice. Given that optimization problems have become increasingly difficult due to high dimensionality and complexity, optimization algorithms have suffered from issues such as stagnation, lack of diversity, and instability. According to literature, large-scale optimization problems typically involve.

- Population coordination.
- Diversity management.
- Multi-population optimization.
- Search with perturbations.
- Operator scheduling.

Such strategies increase the reliability of the search procedure without decreasing its convergence speed.

The findings from our experiments show that benchmark landscapes provide a controlled setting to test operator interaction under different optimization conditions [1, 6]. This is due to the fact that each benchmark landscape focuses on certain problems in the search process.

There is a gap in knowledge concerning the theoretical foundations of operator compatibility, convergence vs. adaptation, and operator coordination. Moreover, issues associated with benchmarking in optimization studies are not yet resolved.

In summary, the recent advancements suggest that the study of numerical optimization has been gradually heading towards more advanced learning-based optimization systems which will be able to coordinate different operator classes based on their landscape and optimization-state [2, 19].

7. Open Research Challenges

Even with the significant improvements obtained from current hybrid metaheuristics, there still exist some major issues that need to be addressed in the field of numerical optimization. The existing optimization

methodologies depend largely on the empirical analysis, manually created hybridization approach, and benchmark-based assessment which means that there is an evident gap in theoretical aspects, reproducibility, scalability, and autonomous coordination.

One of the issues involves the theoretical aspect of operator coordination and adaptation. While there exists an impressive number of hybrid approaches that perform exceptionally well in experiments, there is not much mathematically based information about the following aspects:

- Compatibility of operators.
- Interactions of convergence.
- Adaptive behaviour.
- State transition.

It is also hard to explain how it works due to the black-box nature of the adaptive optimization framework.

Reproducibility and repeatability are common issues in the benchmarking research. Optimizing algorithms usually consider the following issues, among others:

- Parameter configuration.
- Termination criteria.
- Initial configurations.
- Population sizes.
- Objective functions evaluation.

for instance, making it difficult to compare frameworks. Often, the quality of optimization has been determined according to the objective function values.

Scalability has consistently been an unresolved challenge. A considerable number of optimization algorithms operate well on low-dimensional benchmark functions but are unstable, suffer diversity loss, or incur high computational overhead when dimensionality and complexity increase. Accordingly, large-scale optimization has been difficult for:

- High-dimensional landscapes.
- Composition functions.
- Dynamic optimization situations.
- Computationally demanding searches.

Self-adaptive operator coordination is also an important component. Currently, most of the hybrid algorithms use predetermined scheduling and hybridization strategies as opposed to self-adaptation. Designing an algorithm that can identify and adapt the operators for solving problems is not an easy task.

Redundancy of algorithms and too many optimization algorithms also need to be considered by the optimization research community. Many breakthroughs have come about because of metaphors; nevertheless, all the breakthroughs are based on existing operators but slightly modified. In the future, research in optimization should focus on:

- New operators.
- Universal optimization platforms.
- Benchmarking approaches.
- Hybridization strategies.
- Compatibility platforms.

8. Future Directions

Future numerical optimization research is expected to focus increasingly on adaptive and intelligent operator coordination frameworks capable of dynamically restructuring search behavior according to landscape conditions and optimization-state feedback [2, 19]. Rather than relying on fixed hybridization structures, upcoming optimization systems will likely emphasize continuous adaptation, autonomous operator interaction, and learning-assisted search management.

One important direction involves adaptive operator ecosystems in which multiple search operators cooperate dynamically during optimization. Such systems may continuously regulate:

- Exploration intensity.
- Local refinement behaviour.
- Perturbation frequency.
- Diversity maintenance.
- Population coordination.

According to the convergence progress and landscape characteristics. Recent advances in:

- Hyper-heuristics.
- Reinforcement-learning-guided operator selection.
- Adaptive scheduling frameworks.

- Machine-learning-assisted optimization.

Suggest that future optimizers may increasingly perform autonomous search coordination without relying entirely on manually designed hybridization rules.

The future research is anticipated to focus on developing modular and reusable optimization frameworks with interchangeable operators. With the help of modular design, the following will be enabled.

- Reusability of operators' libraries.
- Adaptive composition of operators.
- Automatic design for hybridization.
- Large-scale testing of the compatibility of the framework.
- Large-scale experimentation.

This strategy will significantly increase reproducibility of the results and prevent duplication of efforts in the development of new optimization strategies.

Another emerging direction involves tighter integration between optimization systems and intelligent learning mechanisms. Future optimization frameworks may increasingly combine.

- Surrogate-assisted optimization.
- Predictive learning.
- Adaptive landscape analysis.
- Optimization of memory systems.
- Search-state prediction.
- Data-driven operator scheduling.

Such approaches can contribute significantly to the efficiency of optimization processes in many difficult cases, including.

- Extremely complex problems.
- Dynamic optimization environment.
- Huge search space.
- Cases of optimizing under uncertainty.

Moreover, based on existing research, we may expect the future development of optimization systems toward the integration of these systems within self-controlled computing infrastructures. These are, for instance, such systems as:

- Decision-making systems.
- Cyber-physical infrastructures.
- Simulation systems.
- Ecosystems of optimization.

For such types of systems, optimization algorithms will operate constantly, adjusting to the changes in the environment. In such a case, it is required to provide the ability of constant adjustment of the process, synchronization of search process and autonomous decision-making about the algorithm's behaviour.

Ultimately, numerical optimization research appears to be progressing toward self-evolving optimization systems capable of continuously learning, reorganizing, and improving their own search strategies across multiple landscape categories and optimization conditions.

9. Conclusion

This paper provides an analysis of hybrid metaheuristics based on operators that work on numerical optimization landscapes. Specific types of operator combinations that are usually used for exploration, exploitation, diversification, perturbation, and adaptation of hybrid metaheuristics are identified. It is shown that some types of landscapes impose certain requirements on operators' cooperation, depending on modality, separability, dimensionality, and optimization difficulty. Thus, it proves that it is crucial to consider the problem of hybridization from the viewpoint of a landscape when dealing with modern numerical optimization problems. Finally, one can argue that future optimization systems will apply the techniques of adaptive coordination, intelligent operator choice, and self-evolutionary search in order to adjust their optimization activity based on the current landscape characteristics.

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